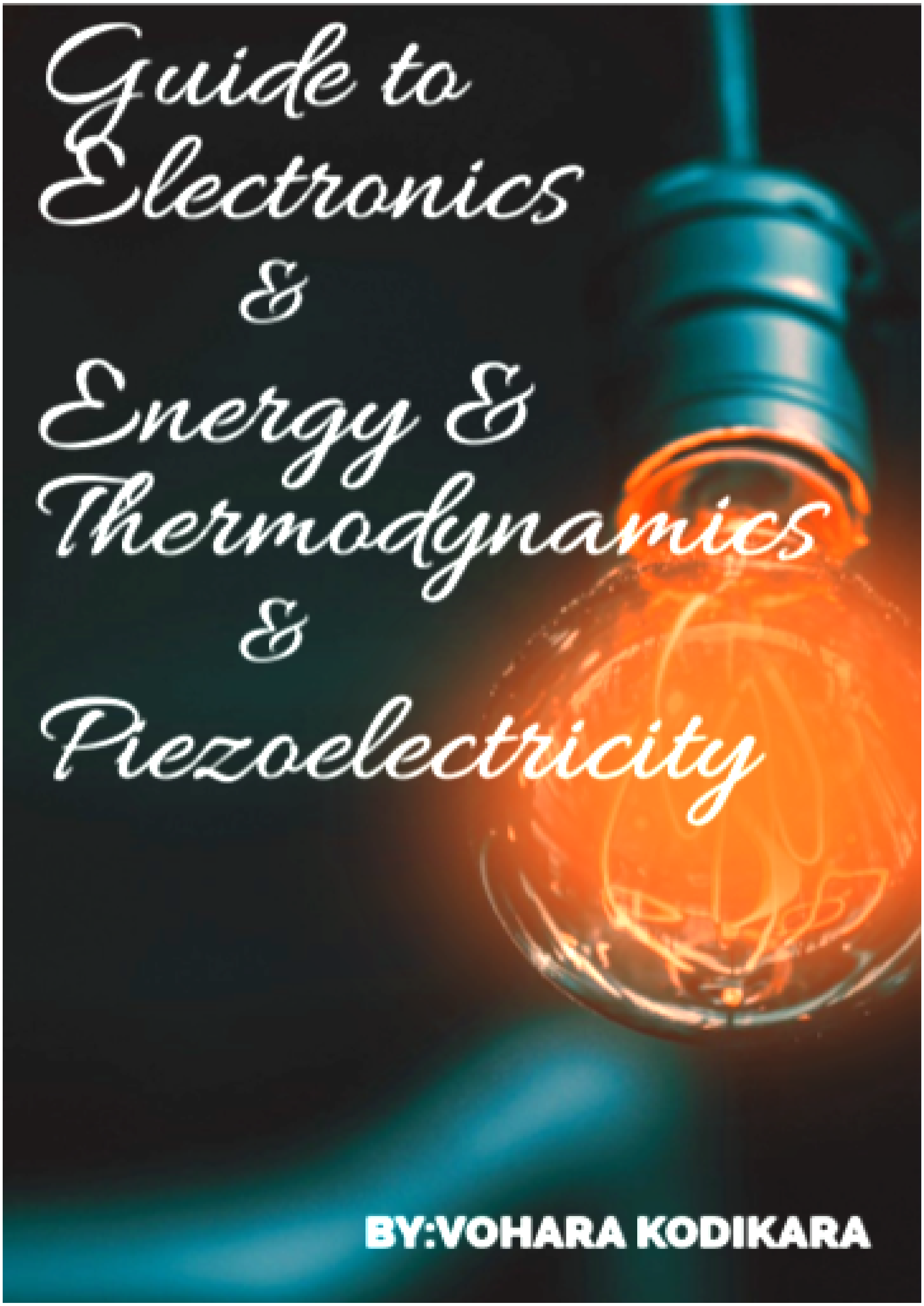
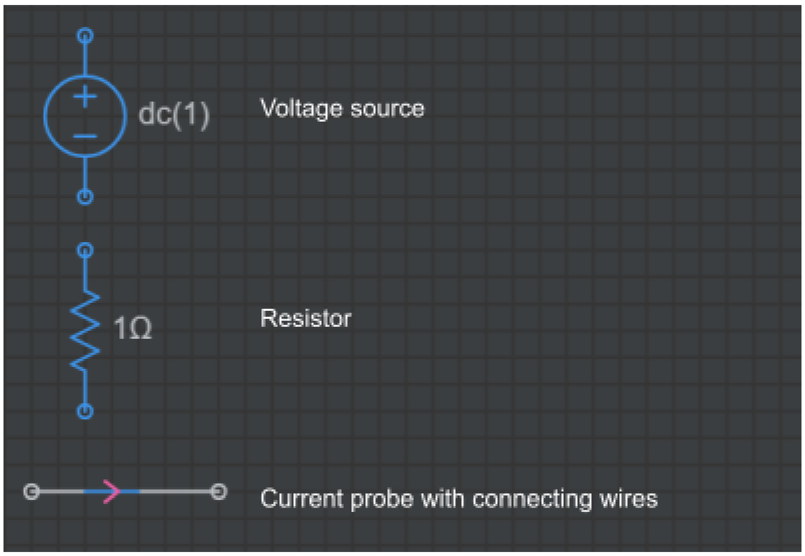

Guide to Electronics
&
Energy &
Thermodynamics
&
Piezoelectricity
BY:VOHARA KODIKARA

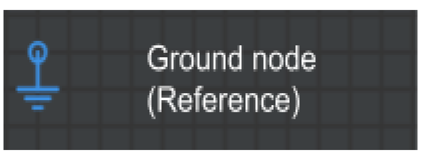
Electronics

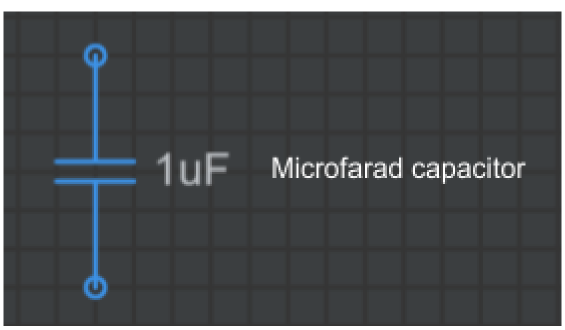
Contents

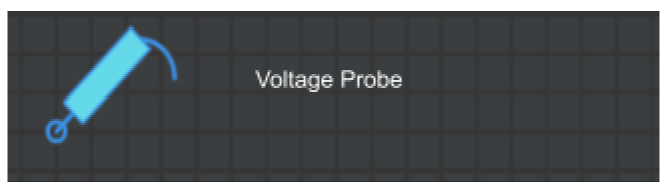
Electrical components

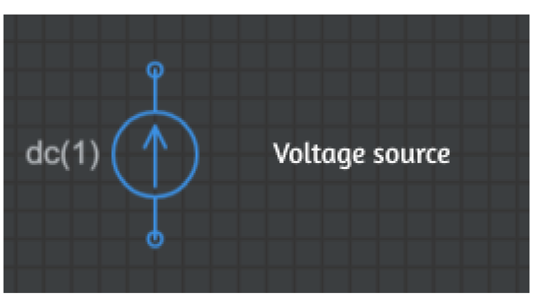
We will be building circuits using the circuit sandbox software.











Important suffixes

The table below shows some important suffixes used in physics for quantities

Suffix	Mutlipier
T	10^{12}
G	10^{9}
M	10^{6}
K	10^{3}
m	10^{-3}
u	10^{-6}
n	10^{-9}
p	10^{-12}
f	10^{-15}

DC Analysis

DC (Direct Current) Analysis is a fundamental technique used in electrical engineering to understand the behavior of electronic circuits when a constant voltage or current source is applied.

1. Steady State: In DC analysis, circuits are analyzed under steady-state conditions where voltages and currents do not change over time (t=0)

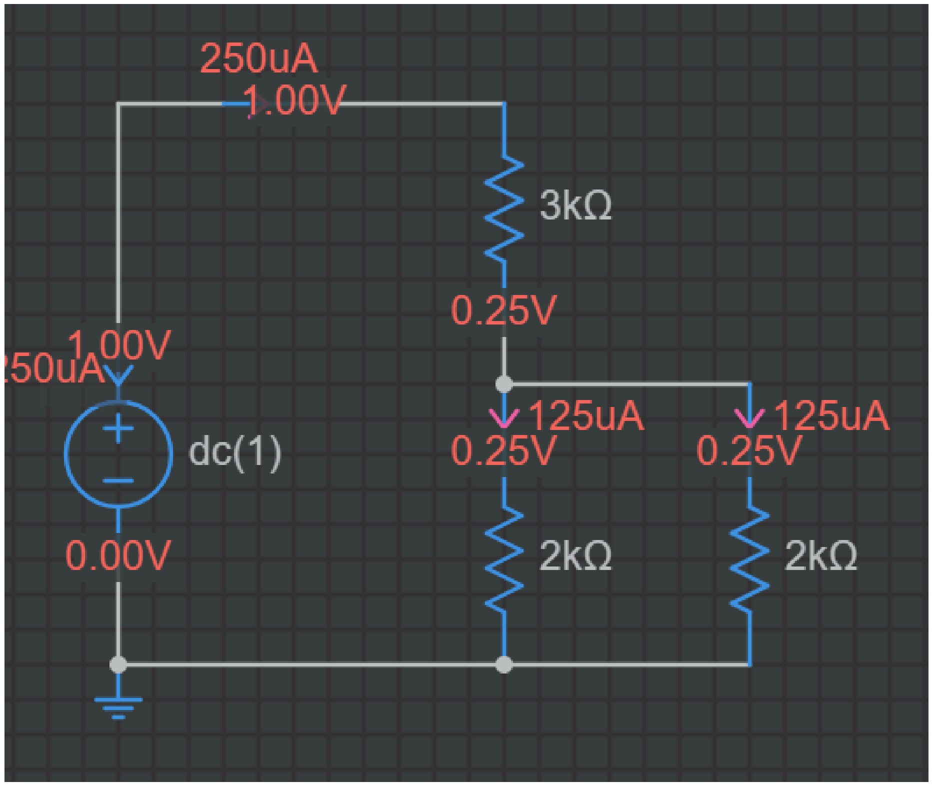
2. Resistors: The primary components considered are resistors, as capacitors and inductors behave like open and short circuits, respectively, in the DC steady-state.

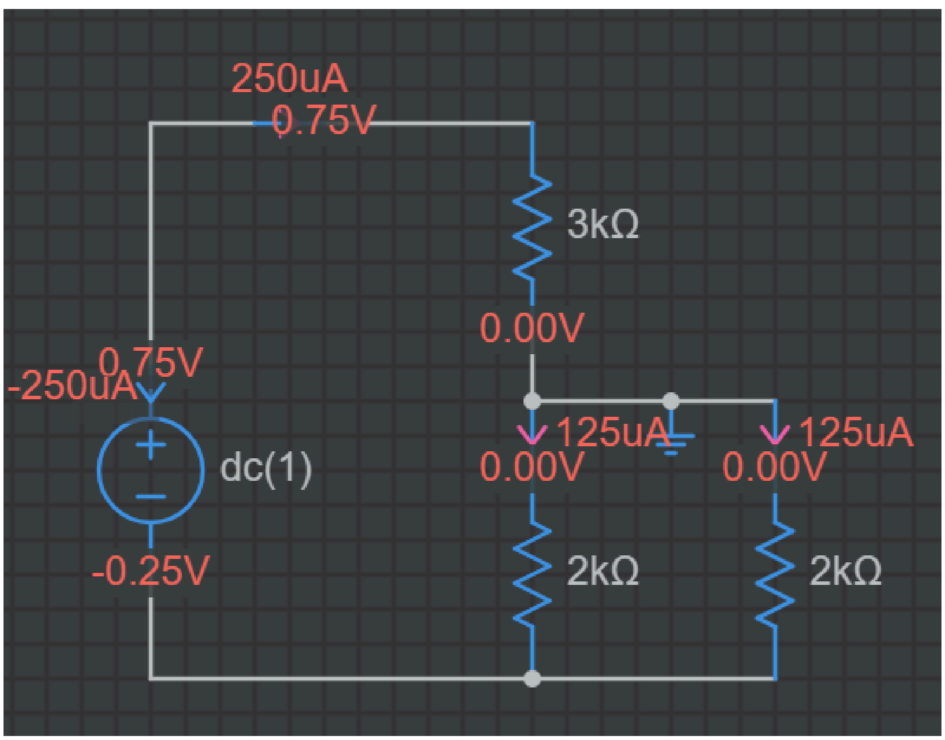
3. Ohm's Law: The relationship between voltage (V), current (I), and resistance (R) is governed by Ohm's Law: V=IR

4. Kirchhoff's Laws:
   - Kirchhoff's Voltage Law (KVL): The sum of all voltages around a closed loop is zero.
   - Kirchhoff's Current Law (KCL): The sum of all currents entering a junction equals the sum of all currents leaving the junction.

5. The voltage values are given relative to the indicated reference node (called 'ground')

Reference Node: A reference node, or ground, is a point in a circuit with zero voltage. It acts as a common point for measuring all other voltages in the circuit.



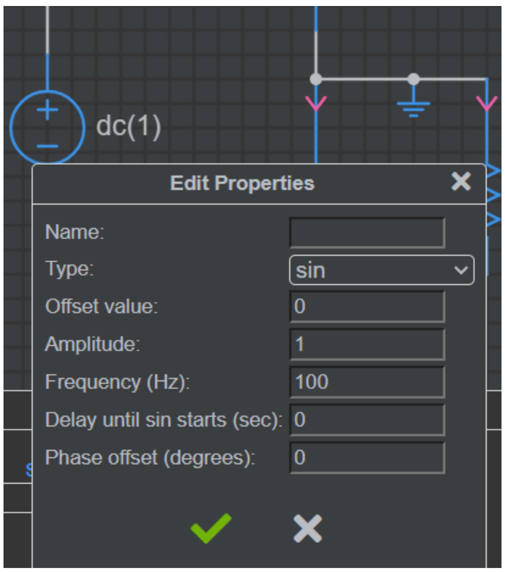


(Example images of a DC analysis, when the reference node is in two different places)

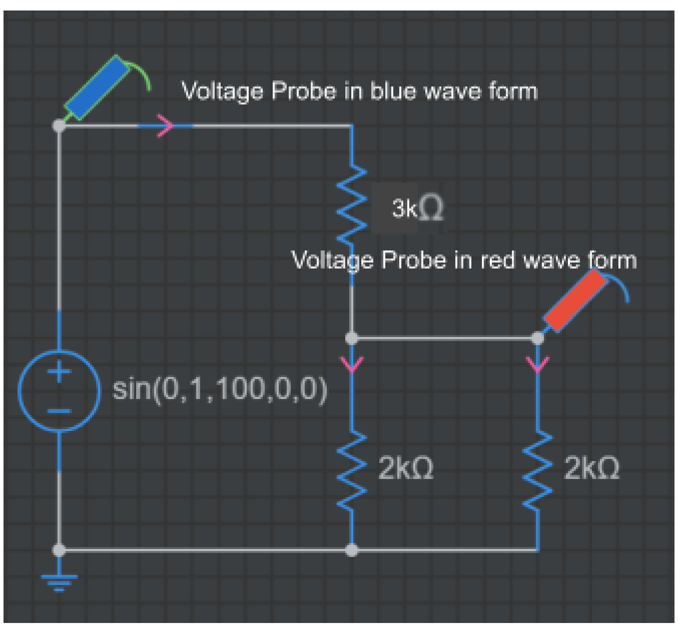
Transient Analysis

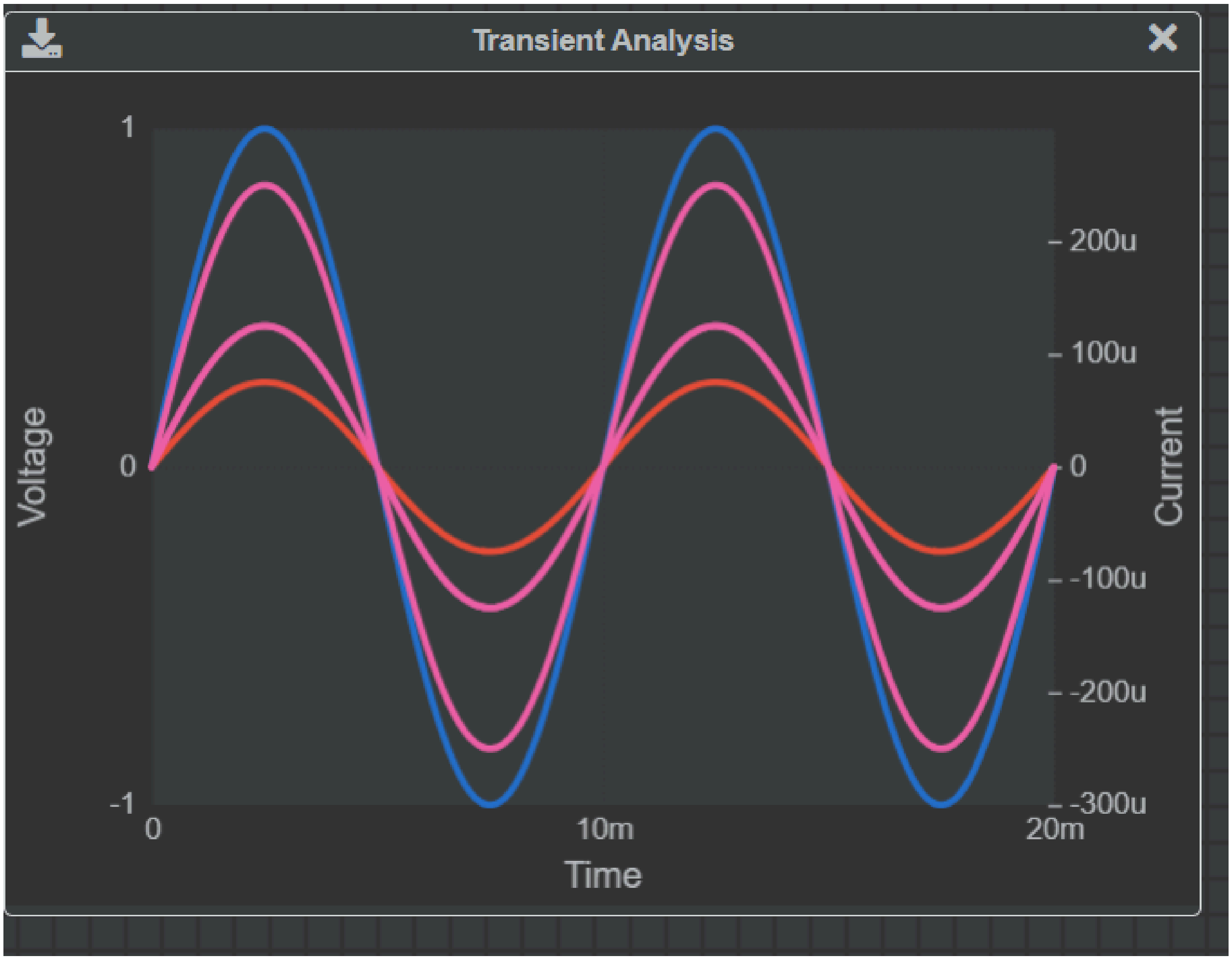
Transient Analysis examines how electrical circuits respond to changes in voltage or current over time. We can add voltage or current probes into the circuit to view plots of V and I versus time.

Voltage Probe: A voltage probe is a tool used to measure the electrical potential difference between two points in a circuit.



(shows the image where we will be changing the DC to a sinusoidal wave with a frequency of 100 Hertz)





The larger blue wave is from the blue-form voltage probe. The voltage form in blue form is connected to measure the voltage of the battery and the voltage probe in red form (smaller red wave) shows a quarter of the voltage of the battery.

The time measured in the above graph is 20 milliseconds.

The pink waves show the current over time from the current probes. These current probes are marked with pink arrows in the circuit diagram.

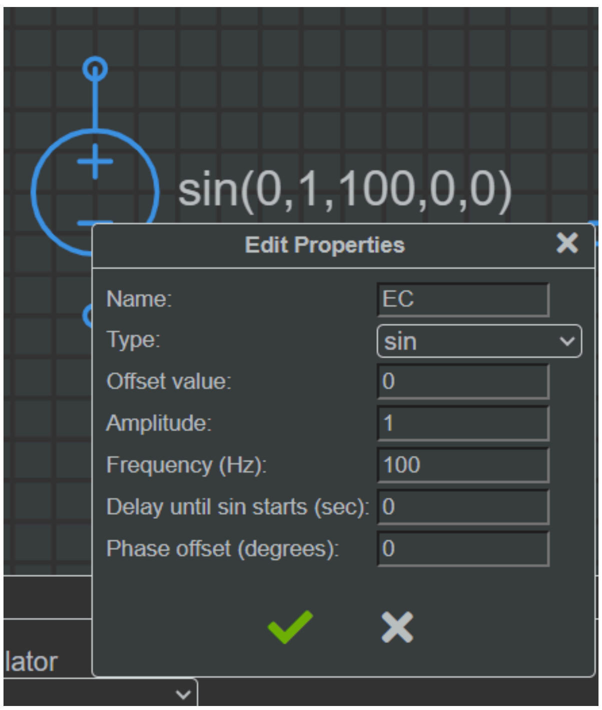
We can add more voltage and current probes and see the current and voltage changes over time on the same plot. We can also set the time value to any time and measure the changes over time.

AC Analysis

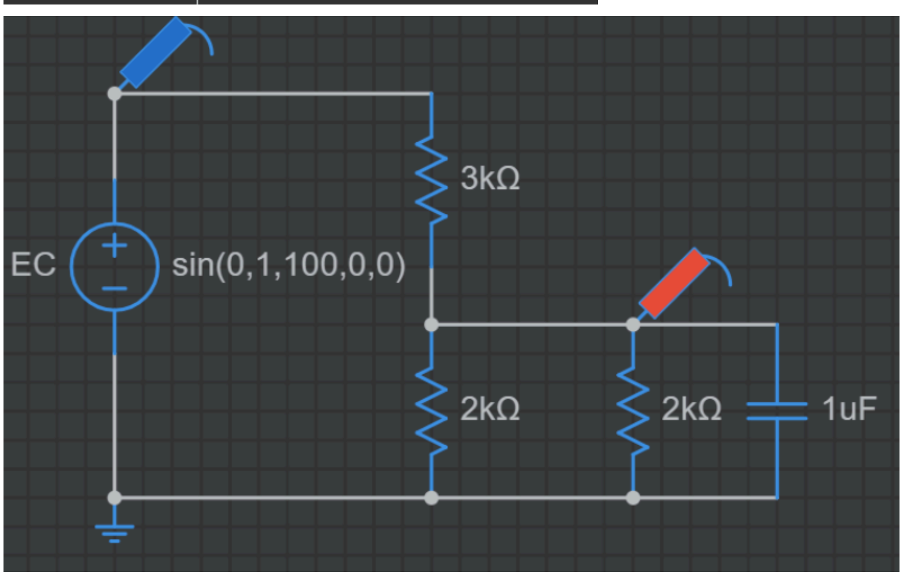
It calculates the circuit's frequency response by injecting a sine wave at a particular point in the circuit and then we can use voltage probes to see the response of the circuit at that point.

This helps us understand how a circuit behaves with different frequency signals.
This analysis helps to observe changes in amplitude and phase as the frequency of the signal increases.

AC analysis simulation:



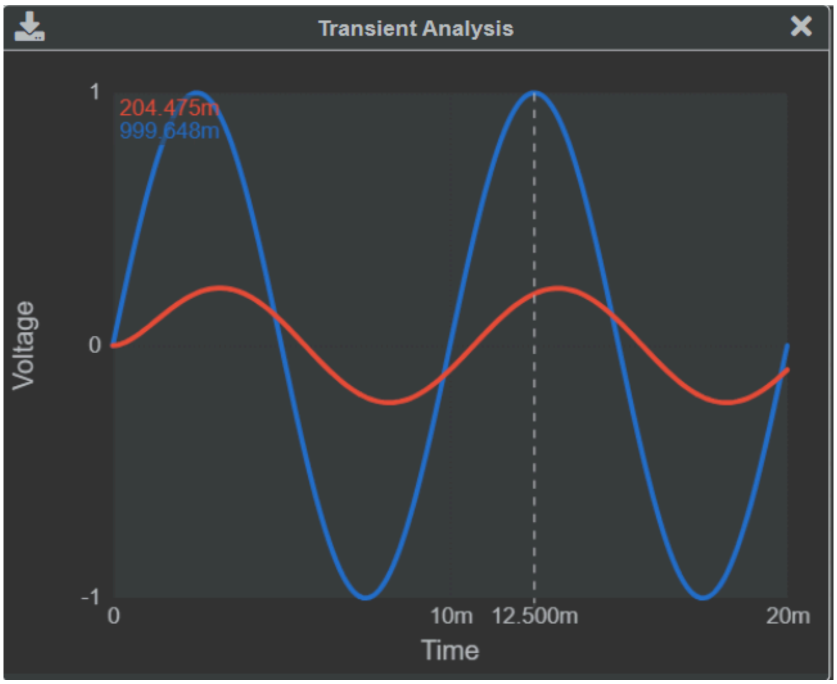
(setting of the voltage source -sinusoidal wave source which is named as EC)



(circuit diagram, with a microfarad capacitor)

A microfarad capacitor is a small device that stores electrical energy. Its storage capacity is measured in microfarads (µF), which is a very small unit. It is used to store electrical energy and release it when needed. It also helps smooth out fluctuations in power and filter signals in electronic devices.

Transient analysis for above circuit diagram (100Hz) :



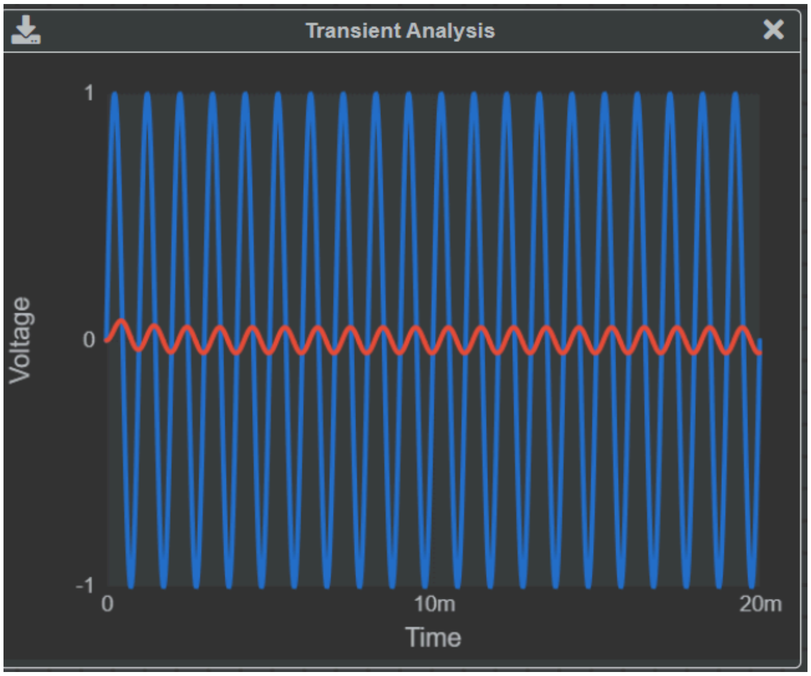
The transient response of the middle note of the voltage divider, which is estimated to be a quarter of the voltage of the source, is a little less than we expected. So if we expected it to be 260 millivolts, we see 230 millivolts.

We can also see that the peak of the voltage divider is a little shifted from the peak of the battery waveform.

We can clearly distinguish the difference between a transient analysis with the capacitor and without the capacitor.

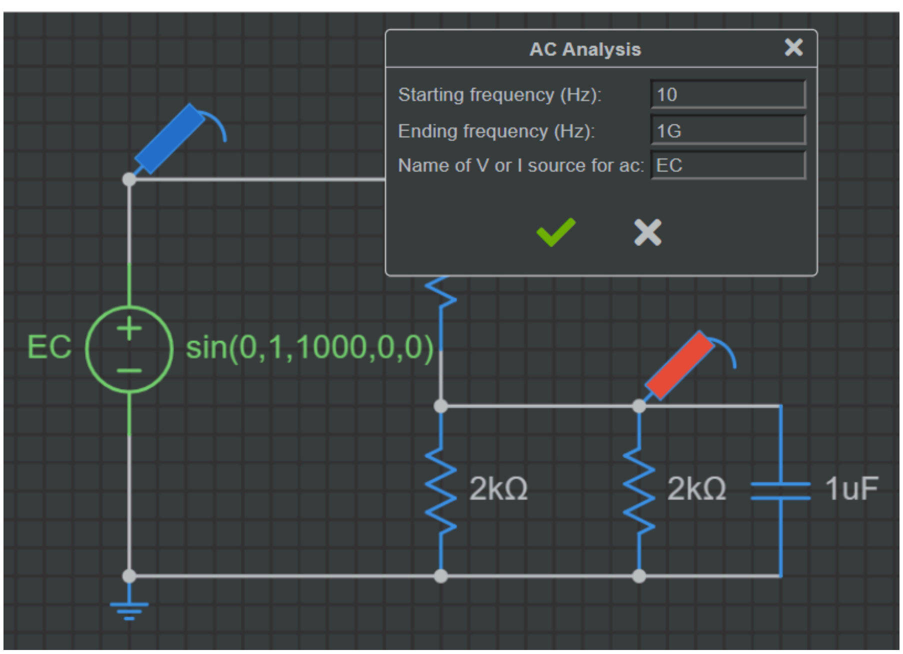
So at 100 Hz the capacitor has made the amplitude to be less and shifted in time (made the response to be changed)

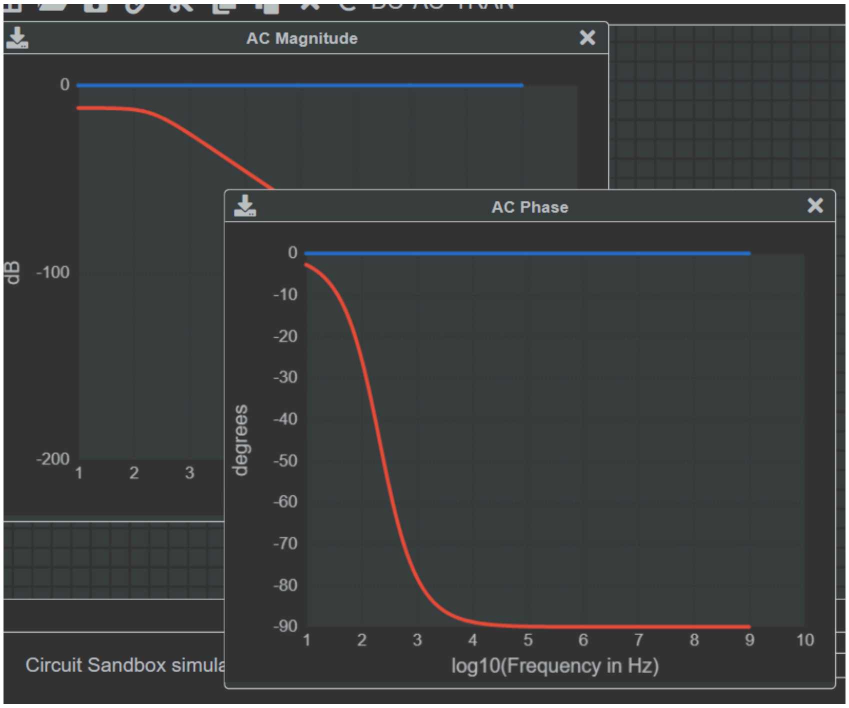
Transient analysis for circuit diagram (1000Hz) :



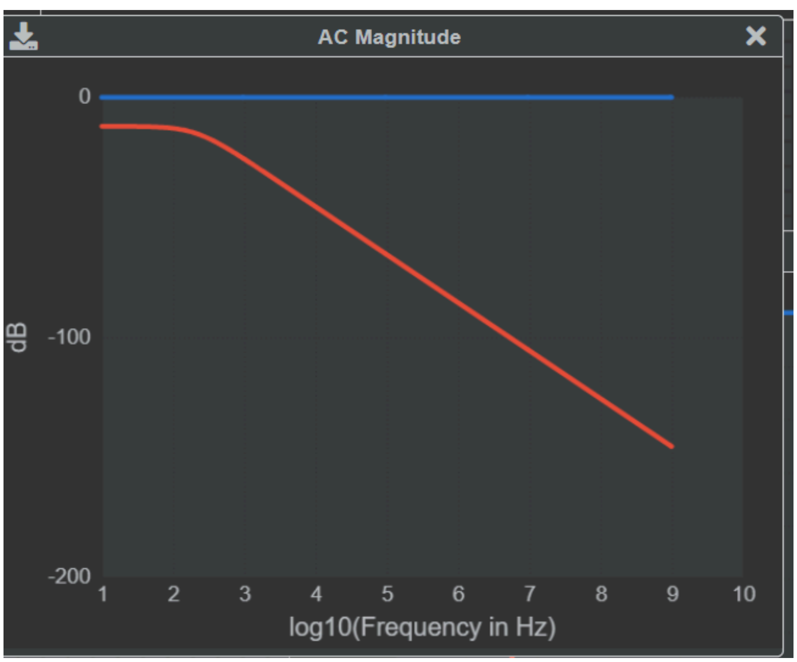
We can see 10 times the number of oscillations and if we look at the red wave form we can see that it is shifted almost 90 degrees relative to the oscillations of the blue wave form.
The response is smaller, which would be around 62 millivolts.

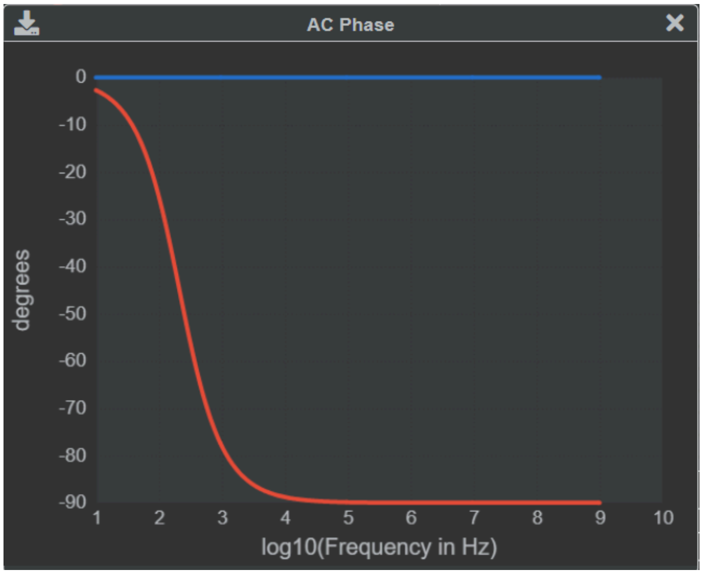
Thus, if we increase the frequency of the test wave, we can see that the response is getting smaller furthermore.
The AC analysis tool helps us to determine that.





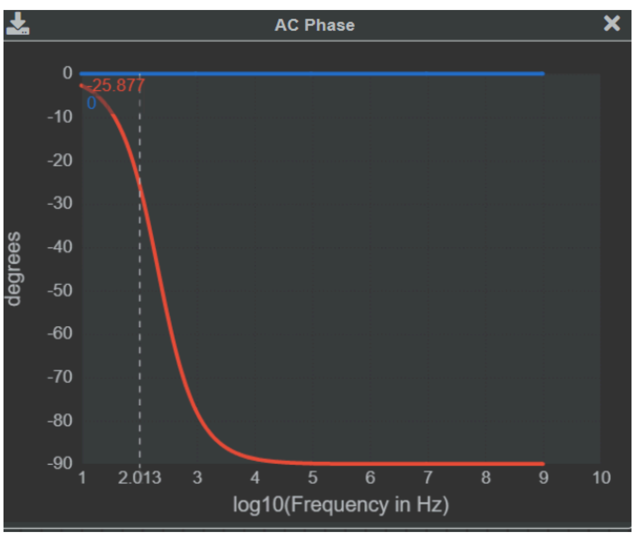
(In AC analysis, we get two windows)

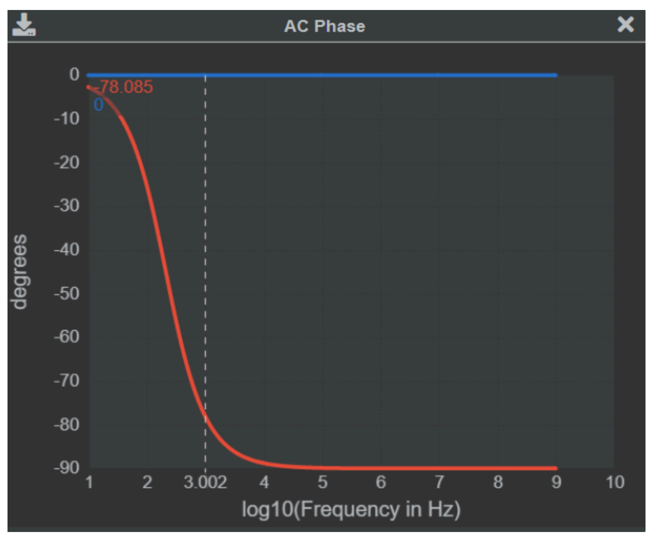




The frequency is plotted in the log scale. So 100 Hz is 10 to the 2 while 1000Hz is 10 to the 3.

When we look at the AC phase graph, we can see that there is a phase change of about 25 degrees when 100 Hz and we can also see a shift in phase of almost 90 degrees at 1000Hz.





First-Order Differential Equation

Linear first-order constant coefficient Differential Equations :

What does it mean?

An equation form for a Linear first-order constant coefficient Differential Equation :

$\frac{dy}{dx} + by = a$, where $\frac{dy}{dx}$ is the derivative, y is an unknown function and b and a are constants.

These types of equations describe how the rate of change of a function is related to the function itself.

$Ax' + Bx = f(t)$, this is first order because there is only one derivative. And constant coefficient because the coefficients of x and x prime are A and B where both are constants. $f(t)$ will always be a known function like e to the 2t etc.

Let's name this equation as I.

The superposition principle states that if two or more functions are solutions to a linear equation, their sum is also a solution to that equation.

Say X_p = I

Then: $Ax_p' + Bx_p = f(t)$

And say that X_h is a homogenous solution (solution to the homogenous equation)

$Ax_h' + Bx_h = 0$ (In the context of differential equations, a homogeneous differential equation has all its terms involving the dependent variable or its derivatives, with no constant terms.)

Hence we can say that $X = X_p + X_h$ is a solution to I

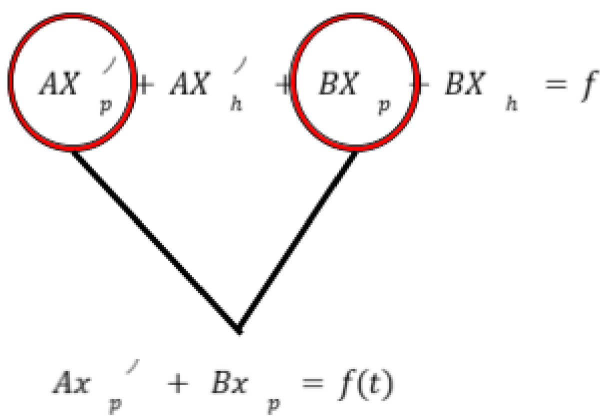
To prove this :

$Ax' + Bx = f$ $(checking\ is\ the\ equal\ sign\ is\ correct)$

$X = X_p + X_h$

$A\left(X_p' + X_h'\right) + B\left(X_p + X_h\right) = f$

$AX_p' + AX_h' + BX_p + BX_h = f$



$f(t) + AX_h' + BX_h = f$

As, $Ax_h' + Bx_h = 0$

$f(t) = f$ thus this proves the superposition principle.

Specific case :

$x' + 4x = 3$

Homogeneous :

$x' + 4x = 0$

$x' = -4x$ (also referred to as $\frac{dx}{dt} = -4x$)

The equation $\frac{dx}{dt} = -4x$ represents an exponential decay because the rate of change of xxx is proportional to x itself, and it is decreasing over time.

$\frac{dx}{x} = -4dt$

Integrate :

$\int \frac{1}{x} = \int -4dt$

This gives :

$ln(x) = -4t + c$

So:

$x = e^{-4t+C}$

$x = e^{-4t} \times e^{C}$

Let e^{C} be a constant called ‘a’:

$x = e^{-4t} \times a$

**$x_h = ae^{-4t}$

Particular :

(in this case, we should try and think of a solution)

$X_p = A$

Substitute ;

$x_p' + 4x_p = 3$

$0 + 4A = 3$ (differentiation of a constant gives 0)

$A = \frac{3}{4}$, so $X_p = \frac{3}{4}$

Superposition principle :

$X = X_p + X_h$

$X = \frac{3}{4} + a\mathrm{e}^{-4t}$

(A homogeneous differential equation is one where there is a combination of functions of x and of its derivatives x', x'' etc..., in the left-hand side, and there is a 0 in the right-hand side.)

Homogeneous first-order linear ordinary differential equations :

Homogeneous linear ordinary equation :

$v' + a \times v = 0$

$v' = - a \times v$

$\frac{dv}{v} = - a \times dt$

Integrate :

$$\int \frac{dv}{v} = \int - a \times dt$$

This gives :

$$ln(v) = - a \times t + c$$

Let e ^{C} be a constant called 'c':

$$v = e^{-at} \times c$$

So :

$$v' = - a \times e^{-at} \times c$$

$$v' + a \times v = - a \times e^{-at} \times c + a \times e^{-at} \times c = 0$$

Inhomogeneous first-order linear ordinary differential equations:

Inhomogeneous linear ordinary equation :

$$v' + a \times v = q(t)$$

We have to multiply both sides of the equation by $e^{\int adt}$

$$\frac{d}{dt}\left(e^{\int adt} + v(t)\right) = q(t) \times e^{\int adt}$$

Differentiate and integrate :

$$e^{\int adt} v(t) = \int (q(t) \times e^{\int adt} \times dt) + c$$

Isolating v gives :

$$v(t) = e^{-\int a dt}\left[\int (q(t) \times e^{\int a dt} \times dt) + c\right]$$

Second-order linear homogeneous differential equations

$$a(x)y'' + b(x)y' + c(x)y = d(x)$$

This is second order, because the second derivative is the highest derivative. It is linear because we have functions of x.

Homogeneous :
$Ay'' + By' + Cy = 0$ $(homogeneous\ as\ it\ is\ equal\ to\ 0)$

Say $g(x)$ is a solution :
$Ag'' + Bg' + Cg = 0$

So is a constant (D) times $g(x)$:
D, $g(x)$?

$ADg'' + BDg' + CDg = 0$
$D(Ag'' + Bg' + Cg) = 0$
So :
$D \times 0 = 0$

So yes D, $g(x)$ is a solution.

Say $h(x)$ is also a solution :

$g(x) + h(x)$

$$A(g'' + h'') + B(g' + h') + C(g + h) = 0$$

$$Ag'' + Ah'' + Bg' + Bh' + Cg + Ch = 0$$

$$\left[Ag'' + Bg' + Cg\right] + \left[Ah'' + Bh' + Ch\right] = 0$$

$$0 + 0 = 0$$

Since $g(x)$ *and* $h(x)$ are solutions, the sum of them is also a solution.

Example 1 (specific case) :

$y'' + 5y' + 6y = 0$

$y = e^{rx} \leftarrow$ (assumption)

(r is a constant)

$\Downarrow$

$y' = re^{rx}$

$y'' = r^2e^{rx}$

} Substitute

$r^2e^{rx} + 5re^{rx} + 6e^{rx} = 0$

$e^{rx}(r^2 + 5r + 6) = 0$

$e^{rx} \neq 0$

(cannot be equal to zero)

$r^2 + 5r + 6 = 0$

$(r+2)(r+3) = 0$

$r = -2$, $r = -3$

$y_1 = e^{-2x}$

$y_2 = e^{-3x}$

* we learnt that a constant times a solution is also a solution.

$C \Rightarrow$ constant

$y_1 = C_1e^{-2x}$

$y_2 = C_2e^{-3x}$

} more general solutions

* If we have 2 different solutions and we add them, we get a solution.

$y(x) = C_1e^{-2x} + C_2e^{-3x} \Rightarrow$ general solution to the differential equation.

Now, we will find a particular solution ?

Initial conditions given :

$y(0) = 2$

$y'(0) = 3$ } → for $y'' + 5y' + 6y = 0$

Solve for c_1 and c_2 ?

- - - - - - -

$y(0) = 2$

General solution is : $y(x) = c_1 e^{-2x} + c_2 e^{-3x}$

$$y(0) = c_1 e^{-2\times 0} + c_2 e^{-3\times 0}$$

$$\boxed{2 = c_1 + c_2}$$

$$y'(x) = -2c_1 e^{-2x} - 3e^{-3x} c_2$$

$$y'(0) = -2c_1 e^{-2(0)} - 3e^{-3(0)} c_2$$

$$y'(0) = -2c_1 - 3c_2$$

$$\therefore \boxed{-2c_1 - 3c_2 = 3}$$

$$2 = c_1 + c_2 \longrightarrow \times (-2)$$

$$-2c_1 - 2c_2 = -4 \longrightarrow (3)$$

$$-2c_1 - 3c_2 = 3 \longrightarrow (2)$$

$$-3c_2 + 2c_2 = 3 + 4$$

$$-c_2 = 7$$

$$\boxed{c_2 = -7}$$

$$c_1 - 7 = 2$$

$$\boxed{c_1 = 9}$$

Particular solution $= 9e^{-2x} - 7e^{-3x}$

Example 2 (specific case) :

$$4y'' - 8y' + 3y = 0 \qquad y(0) = 2, \quad y'(0) = \frac{1}{2}$$

$$4r^2 - 8r + 3 = 0$$

↑ characteristic equation.

$$r = \frac{8 \pm \sqrt{64 - (4 \times 4 \times 3)}}{2(4)}$$

$$r = \frac{8 \pm \sqrt{16}}{8}$$

$$r = \frac{8 \pm 4}{8}$$

$$r = 1 \pm \frac{1}{2}$$

$$r = \frac{3}{2}, \qquad r = \frac{1}{2}$$

$$y = c_1 e^{\frac{3}{2}x} + c_2 e^{\frac{1}{2}x} \quad \leftarrow \text{general solution}$$

$$y' = \frac{3}{2} c_1 e^{\frac{3}{2}x} + \frac{1}{2} c_2 e^{\frac{1}{2}x}$$

$$y(0) = 2$$

$$2 = c_1 + c_2 \longrightarrow (1) \times \frac{3}{2}$$

$$y'(0) = \frac{1}{2}$$

$$\frac{1}{2} = \frac{3}{2} c_1 + \frac{1}{2} c_2 \longrightarrow (2)$$

$$\frac{3}{2} c_1 + \frac{3}{2} c_2 = 3 \longrightarrow (3)$$

$$c_2 = 3 - \frac{1}{2} \qquad c_1 = 2 - \frac{5}{2}$$

$$c_2 = \frac{5}{2} \qquad c_1 = -\frac{1}{2}$$

Particular solution

$$y = -\frac{1}{2} e^{\frac{3}{2}x} + \frac{5}{2} e^{\frac{1}{2}x}$$

Complex Numbers

Complex Arithmetic

Complex numbers help us to get the square roots of negative numbers.
Complex numbers are in the form of $a + jb,\ where\ a\ and\ b\ are\ constants.$

Examples : (Z is used to denote complex numbers)

$Z = 2 + 3j$ ($called\ complex\ numbers$)

$Z = 3j$ ($called\ pure\ imaginary$)

$Z = 3 = 3 + 0j$ ($called\ real\ number$)

(j is just a number: j = $\sqrt{-1}$, so $j^2 = -1$) –always

$Z = 2 + 3j$, here the real part of Z is 2 and the imaginary part is 3
We show this like :
Re(z) = 2
Im(z) = 3

If we have $(2 + 3j)(4 + 5j)$:

The answer is :
$\left(8 + 10j + 12j + 15j^2\right)$
Since, $j^2 = -1$

$8 + 10j + 12j + 15(-1) = -7 + 22j$

If we have a division, $\frac{4+5j}{2-3j}$ we should multiply the top and bottom by $2 + 3j$

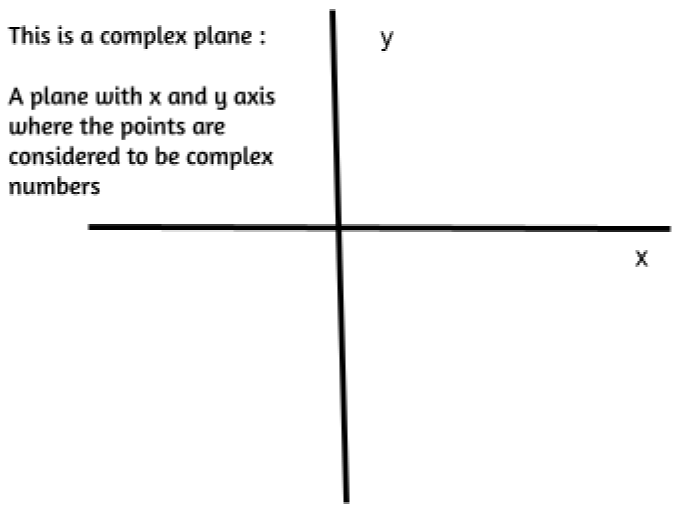
So we get $\frac{-7}{13} + \frac{22j}{13}$

Conjugate of Z/ Z conjugate / Z bar :

$$Z = a + jb \text{ ---------> } \overline{Z} = a - jb$$

$$Z \times \overline{Z} = (a - jb)(a + jb) = a^2 + b^2 = |z|^2$$

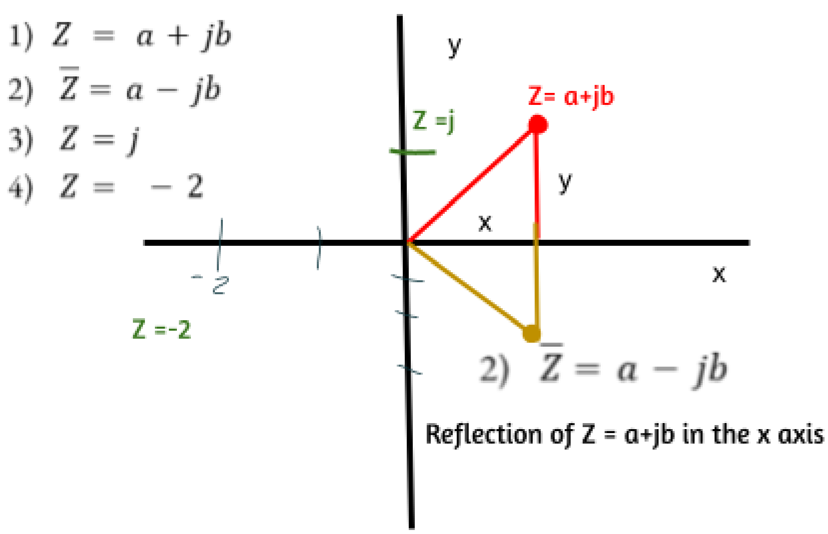
<u>Complex Geometry</u>

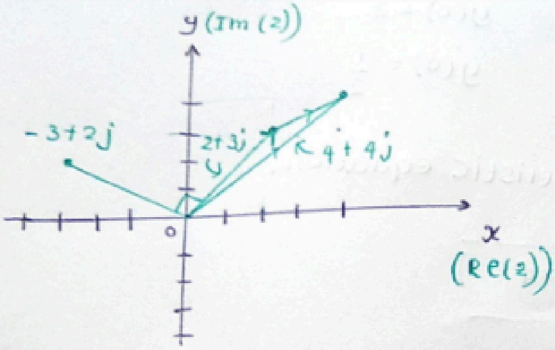


Plotting :

1) $Z = a + jb$
2) $\overline{Z} = a - jb$
3) $Z = j$
4) $Z = -2$

The x-axis is the real axis while the y-axis is the imaginary axis. This is because the values plotted in the x-axis are real and the values in the y-axis are imaginary.





+ : (addition is same as vector addition)

eg : $(2+3j)+(2+j)$

$= 4+4j$

− : (Subtraction is same as vector subtraction)

Example of complex Multiplication : (fully understanding needs Euler's formula)

$j(2+3j)$ $\quad (j^2=-1)$

$= 2j + 3j^2$

$= -3 + 2j$

Magnitude :

$|jz| = \sqrt{(-3)^2+2^2} = \sqrt{13}$ * Multiplying by j did not change the magnitude.

$|z| = \sqrt{2^2+3^2} = \sqrt{13}$

Direction (angle) :

Angle :

Angle between these 2 vectors is $\frac{\pi}{2}$

slope of $\overrightarrow{OJZ} = -\frac{2}{3}$

slope of $\overrightarrow{OZ} = \frac{3}{2}$

$-\frac{2}{3} \times \frac{3}{2} = -1$ (They are perpendicular)

× by j = rotation by $\frac{\pi}{2}$ in a counter clockwise direction (anticlockwi-se)

Polar Coordinates.

Polar coordinates are a way of representing points in a plane using the distance from a fixed point (the origin) and an angle from a fixed direction (usually the positive x-axis). Instead of using the Cartesian coordinates (x,y), polar coordinates use (r,θ), where:

- r is the radial distance from the origin to the point.
- θ (theta) is the angle measured counterclockwise from the positive x-axis to the line segment connecting the origin to the point.

For example, a point in Cartesian coordinates (x,y) can be converted to polar coordinates using:

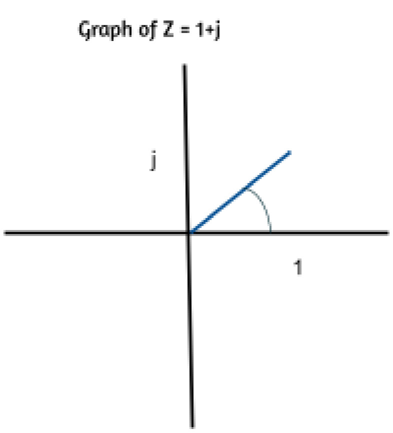
$$r = \sqrt{x^2 + y^2}$$

$$\theta = tan^{-1}\left(\frac{y}{x}\right)$$

A point in polar coordinates (r,θ) can be converted to Cartesian coordinates using the formulas:

$$x = rcos\theta$$

$$y = rsin\theta$$



$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\theta = tan^{-1}\left(\frac{1}{1}\right) = 45\ degrees\ or\ \frac{\pi}{4}$$

If the graph is Z = 2 then r will be 2 and theta value will be 0.

If the graph is Z = j then the r value will be 1 and the theta value will be pi over 2 (90 degrees)

There also can be negative values.

The argument of a complex number is the angle that the complex number makes with the positive real axis in the complex plane.

For a complex number $Z = y + vj$ where y is the real part and v is the imaginary part, the argument (often denoted as $arg(Z)$) can be found using the arctangent function:

$$arg(Z) = tan^{-1}\left(\frac{v}{y}\right)$$

However, because this approach only works correctly in the first and fourth quadrants, we use the atan2 function to get the correct angle for all quadrants:

$arg(Z) = atan2(v, x)$ this argument is usually given in radians from $-\pi\ to\ \pi$

Example :

For the complex number $Z = -1 - i$:

1. Real part $x = -1$
2. Imaginary part $y = -1$

Using $atan2$:

$arg(Z) = atan2(-1, -1) = -135\ degrees$, this places the complex number in the third quadrant correctly.

Euler's Formula.

Euler's formula : $e^{j\theta} = \cos\theta + j\sin\theta$

1) Exponents law :

$$e^{j(\alpha+\beta)} = e^{j\alpha} \cdot e^{j\beta}$$

2) complex exponential differentiation :

$$\frac{d}{d\theta} e^{j\theta} = je^{j\theta}$$

Example :

① $e^{j\frac{\pi}{2}} = \overset{0}{\cos\left(\frac{\pi}{2}\right)} + j\overset{1}{\sin\left(\frac{\pi}{2}\right)} = j$

② $e^{j\pi} = \overset{-1}{\cos(\pi)} + j\overset{0}{\sin(\pi)} = -1 \rightsquigarrow e^{j\pi} = -1$

$(e^{j\pi} + 1 = 0)$

③ $e^{j\cdot 2\pi} = \overset{1}{\cos(2\pi)} + j\overset{0}{\sin(2\pi)}$

$= 1 + 0 = 1$

④ $\sqrt{2}\, e^{j\pi} = \sqrt{2}\cos(\pi) + j\sqrt{2}\overset{0}{\sin(\pi)}$

$= -\sqrt{2} + 0$

$= -\sqrt{2}$

$z = x + jy$ r, θ - polar coordinates.

$$z = x + jy$$

cartesion coordinates

↓ change to polar coordinates

$$x = r\cos\theta$$
$$y = r\sin\theta$$

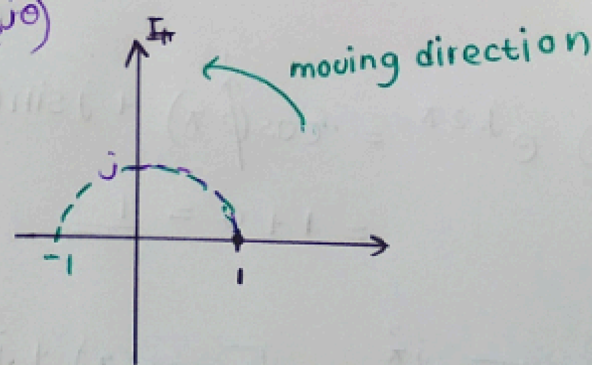
$z = r\cos\theta + r\sin\theta j$ ← Euler's formula.

$$z = r(\cos\theta + j\sin\theta)$$

$z = re^{j\theta}$ ← polar form

Examples: $(re^{j\theta})$

$e^{j\frac{\pi}{2}} = ?$



~~rj~~ = 1 = r

$\therefore r = 1$

$\theta = \frac{\pi}{2}$

$$e^{j\frac{\pi}{2}} = j$$

$e^{j\pi} = ?$

$r = 1$

$\theta = \pi$

$$e^{j\pi} = -1$$

Multiplication using polar coordinates :

$$Z1 = r1 \times e^{j\theta 1}$$

$$Z2 = r2 \times e^{j\theta 2}$$

$$Z1 \times Z2 = r1 \times r2 \times e^{j(\theta 1+\theta 2)}$$

Lengths multiply while angles add.

Division using polar coordinates :

$$Z1 = r1 \times e^{j\theta 1}$$

$$Z2 = r2 \times e^{j\theta 2}$$

$$\frac{r1 \times e^{j\theta 1}}{r2 \times e^{j\theta 2}} = \frac{r1}{r2} e^{j(\theta 1-\theta 2)}$$

Lengths divide while angles subtract.

Inverse Euler and complex Exponentials

Inverse Euler:

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}, \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Proof NOTE: $e^{-j\theta} = \cos\theta - j\sin\theta$

* $e^{j\theta} + e^{-j\theta} = (\cos\theta + j\sin\theta) + (\cos\theta - j\sin\theta) = 2\cos\theta$

$$2\cos\theta = e^{j\theta} + e^{-j\theta}$$

$$\left(\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}\right)$$

* $e^{j\theta} - e^{-j\theta} = \cos\theta + j\sin\theta - (\cos\theta - j\sin\theta)$

$$= 2j\sin\theta$$

$$2j\sin\theta = e^{j\theta} - e^{-j\theta}$$

$$\left(\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}\right)$$

1) $$\int \sin(at)\cos(bt)\,dt = \int \frac{(e^{jat} - e^{-jat})}{2j} \times \frac{(e^{jbt} + e^{-jbt})}{2}$$

$$= \int \frac{e^{j(a+b)t} + e^{-j(a-b)t} - e^{-j(a-b)t} - e^{-j(a+b)t}}{2 \times 2j}$$

~~cosθ~~ sinθ

sinθ

$$= \frac{1}{2} \int \left(\sin((a+b)t) + \sin((a-b)t)\right)$$

→

$$= -\frac{1}{2}\left[\frac{\cos((a+b)t)}{(a+b)} + \frac{\cos((a-b)t)}{(a-b)}\right]$$

Integration of sin is negative cos

Trig identity we learned :

$$\sin a \cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$

① $e^{j\alpha} \cdot e^{j\beta} = (\cos\alpha + j\sin\alpha)(\cos\beta + j\sin\beta) \quad (j^2 = -1)$

$= \cos\alpha\cos\beta + j\sin\beta\cos\alpha + j\sin\alpha\cos\beta + j^2\sin\alpha\sin\beta$

~~$= \cos(\alpha+\beta) + j\sin$~~

$= (\cos\alpha\cos\beta - \sin\alpha\sin\beta) + j(\sin\beta\cos\alpha + \sin\alpha\cos\beta)$

$= \cos(\alpha+\beta) + j\sin(\alpha+\beta)$

$= e^{j(\alpha+\beta)}$

② $\frac{d}{d\theta} e^{j\theta} = \frac{d}{dt}(\cos\theta + j\sin\theta) = -\sin\theta + j\cos\theta$

$je^{j\theta} = j(\cos\theta + j\sin\theta) = j\cos\theta - \sin\theta$

same

power series are consistent :

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$\cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots \quad \text{(even)}$$

$$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots \quad \text{(odd)}$$

$$e^{j\theta} = 1 + j\theta + \frac{j^2\theta^2}{2!} + \frac{j^3\theta^3}{3!} + \frac{j^4\theta^4}{4!}$$

$j^2 = -1$
$j^3 = -j$
$j^4 = 1$

$$e^{j\theta} = 1 + j\theta - \frac{\theta^2}{2!} - \frac{j\theta^3}{3!} + \frac{\theta^4}{4!}$$

$$e^{j\theta} = \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!}\right) + j\left(\theta - \frac{\theta^3}{3!}\right)$$

$\Rightarrow e^{j\theta} = \cos\theta + j\sin\theta$

Complex Replacement

$$A = \int e^{2x} cos(3x)\, dx$$

If we were asked to integrate the above, we can solve it by doing integration by parts twice in calculus. But we can use more complex techniques to solve this :

$J = \int e^{2x} sin(3x)\, dx$, we are introducing another integral here.

$$\int (A + j \times J) = \int e^{2x} cos(3x) + e^{2x} sin(3x) = \int e^{2x} (cos(3x) + jsin(3x))$$

$$\int e^{2x} (cos(3x) + jsin(3x)) = \int e^{2x} e^{j3x} dx$$

$$\int e^{2x} e^{j3x} dx = \int e^{(2+3j)x} dx$$

$$\int e^{(2+3j)x} dx = \frac{e^{(2+3j)x}}{(2+3j)}$$

$$\frac{e^{(2+3j)x}}{(2+3j)} = \frac{e^{2x}(cos(3x)+jsin(3x))}{(2+3j)} \times \frac{2-3j}{2-3j}$$

$$= \frac{e^{2x}}{13} ([2cos(3x) + 3sin(3x)] + j[2sin(3x) - 3cos(3X)])$$

So $A = \frac{e^{2x}}{13} (2cos(3x) + 3sin(3x))$ and $J = \frac{e^{2x}}{13} (2sin(3x) - 3cos(3X))$

Doing the same thing but with polar coordinates:

$A = \int e^{2x} cos(3x)\, dx$

$\tilde{A} = \int e^{2x} e^{j3x} dx = \frac{e^{(2+3j)x}}{(2+3j)}$

$2 + 3j = \sqrt{13} \times e^{j\phi}, \ \phi = angle\ of\ 2 + 3j$

$\tilde{A} = \frac{e^{(2+3j)x}}{\sqrt{13} \times e^{j\phi}} = \frac{e^{2x}}{\sqrt{13}} e^{j(3x-\phi)}$

$\frac{e^{2x}}{\sqrt{13}} e^{j(3x-\phi)} = \frac{e^{2x}}{\sqrt{13}} (cos(3k - \phi) + jsin(3x - \phi))$

$A = \frac{e^{2x}}{\sqrt{13}} cos(3k - \phi)$, this is called the amplitude-phase form. That is we have a changing amplitude, $\frac{e^{2x}}{\sqrt{13}}$, with a phase shift which is represented by phi ϕ.

This is a cosine curve that is shifted by an angle of phi and it is growing exponentially in amplitude.

Complex Roots

Fundamental Theorem of Algebra;

A polynomial of degree n has exactly n complex roots. So a cubic has 3 roots and a quartic has 4 roots.

Example 1:

Find the cube roots of j :

$z^3 = j$ (we need to write j in polar form)

We know that j has a magnitude of 1 so the factor out in the front so the factor in front of the exponential is 1 and we have an angle of pi over 2.

We can add any multiple of 2 pi to the angle of the complex number.

$z^3 = j = e^{(\frac{\pi}{2}+2\pi n)j}$, where n is 1,2,3,.....-1,-2,....

$z = e^{(\frac{\pi}{6}+\frac{2\pi n}{3})j}$, substituting n = 1,2,3 = $e^{\frac{\pi}{6}j}, e^{\frac{5\pi}{6}j}, e^{\frac{9\pi}{6}j}$

$$e^{\frac{\pi}{6}j} = cos(\frac{\pi}{6}) + jsin((\frac{\pi}{6}) = \frac{\sqrt{3}}{2} + \frac{1}{2}j$$

$$e^{\frac{5\pi}{6}j} = cos(\frac{5\pi}{6}) + jsin((\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2} + \frac{1}{2}j$$

$$e^{\frac{9\pi}{6}j} = cos(\frac{9\pi}{6}) + jsin((\frac{9\pi}{6}) = -j$$

Solutions are : $\frac{\sqrt{3}}{2} + \frac{1}{2}j, -\frac{\sqrt{3}}{2} + \frac{1}{2}j, -j$

Example 2:

Find the solutions to $(1 + j)^{\frac{1}{2}}$

$$z^2 = 1 + j = \sqrt{2}e^{j(\frac{\pi}{4}+2n\pi)},\ n = 0, 1, 2$$

$$z = 2^{\frac{1}{4}} e^{j(\frac{\pi}{8}+n\pi)}$$

Solutions :

$$2^{\frac{1}{4}} e^{j(\frac{\pi}{8})},\ 2^{\frac{1}{4}} e^{j(\frac{9\pi}{8})}$$

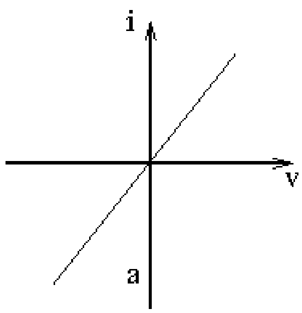
Lumped element abstraction

The lumped element abstraction simplifies electrical circuits by assuming that voltage (V) and current (I) are concentrated in discrete components like resistors, capacitors, and inductors. This model treats wires as having no significant resistance, capacitance, or inductance, making it easier to analyse circuits using basic laws like Ohm's Law (V=IR) and Kirchhoff's Laws. It works well for low-frequency circuits but needs to be more accurate for high-frequency applications where the circuit dimensions are comparable to the signal's wavelength.

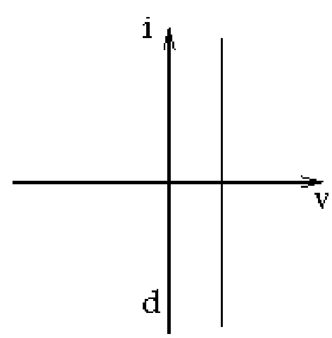
For example, if we were asked to calculate the current flowing through a bulb, we can use a discrete resistor and find the R-value so that V = IR and R is v over I.
Thus R is a lumped element abstraction for the bulb.

v-i graphs:

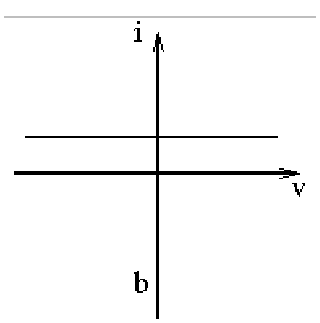
a) Linear resistor:

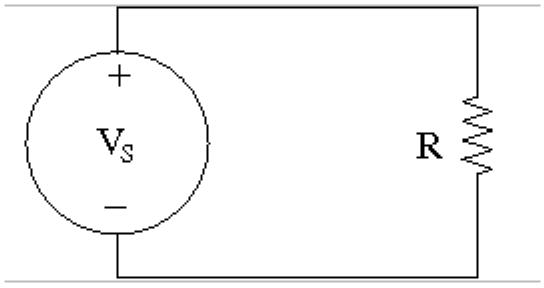


b) independent voltage source:



c) Independent current source :





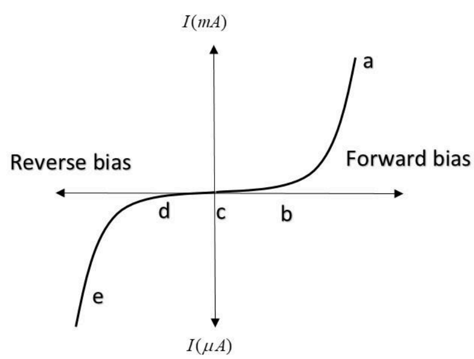
If the voltage of the source is 20 V and the resistance of the resistor is 100Ω, what will be the power dissipated in the resistor and the power entering the source in Watts?

P = VI

So the power dissipated in the resistor would be 4W and the power entering the source would be −4W.

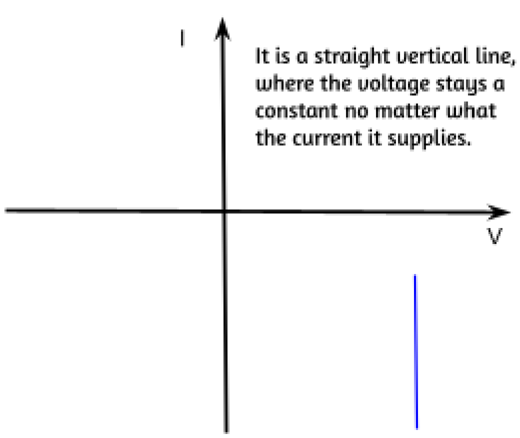
Lumped elements: resistors, fixed resistors, Zener diode, diode, thermistor, photoresistor, dead battery, battery, bub, switches, MOFETs, capacitors, inductors.

v-i graphs for a Zener diode:

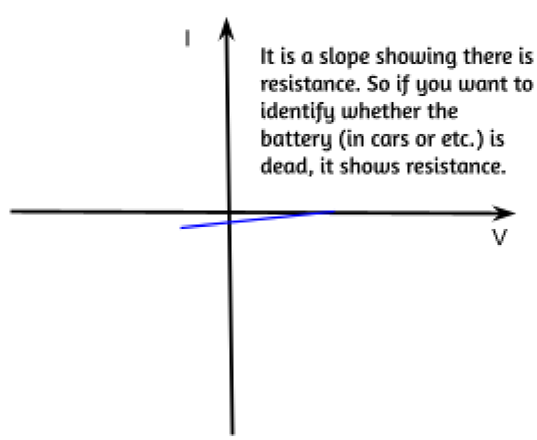


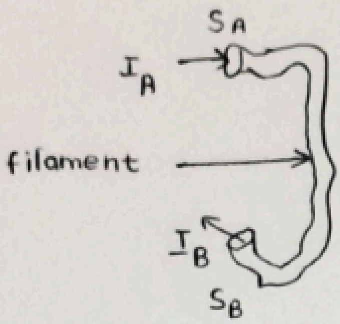
v-i graph for a battery and a dead battery:

Good battery:



Bad battery :





I_A : current going into filament

I_B : current going out of filament

S_A : Surface area A

S_B : Surface area B

Maxwell's Equations :

one of these equations show that the difference in current entering and leaving a volume equals the rate of change of electric charge in the volume.

$$\frac{\delta q}{\delta t}$$

Current density (J) : How much electric current flows through a unit area of surface.

To find current going into S_A and S_B, we integrate the current density over thes surfaces.

$$\underbrace{\int_{S_A} J.\, dS_A}_{I_A} - \underbrace{\int_{S_B} J.\, dS}_{I_B} = \frac{dq}{dt}$$

$$I_A - I_B = \frac{\delta q}{\delta t}$$

$$\rightarrow I_A = I_B \left(\text{only when } \frac{\delta q}{\delta t} = 0\right)$$

*For current to be uniquely defined (the same current enters and leaves), the rate of change of charge must be zero.

What about voltage ?

v_{AB} defined $\frac{\partial \phi_B}{\partial t} = 0$

so :

$$V_{AB} = \int_{c\,AB} E \cdot dt$$

The voltage depends on the electric field and change in magnetic flux.

Third assumption :

* Signal speeds of interest should be way lower than the speed of light.

Assumptions :

① $\frac{\delta \rho}{\delta t} = 0$ (constant charge)

② $\frac{\delta \phi}{\delta t} = 0$ (constant magnetic flux)

③ signal speed much lower than speed of light.

Lumped Matter Discipline (LMD) or self imposed constraints :

- $\frac{\delta q}{\delta t} = 0$ (inside elements bulb, wire, battery)
- $\frac{\delta \phi_B}{\delta t} = 0$ (outside)

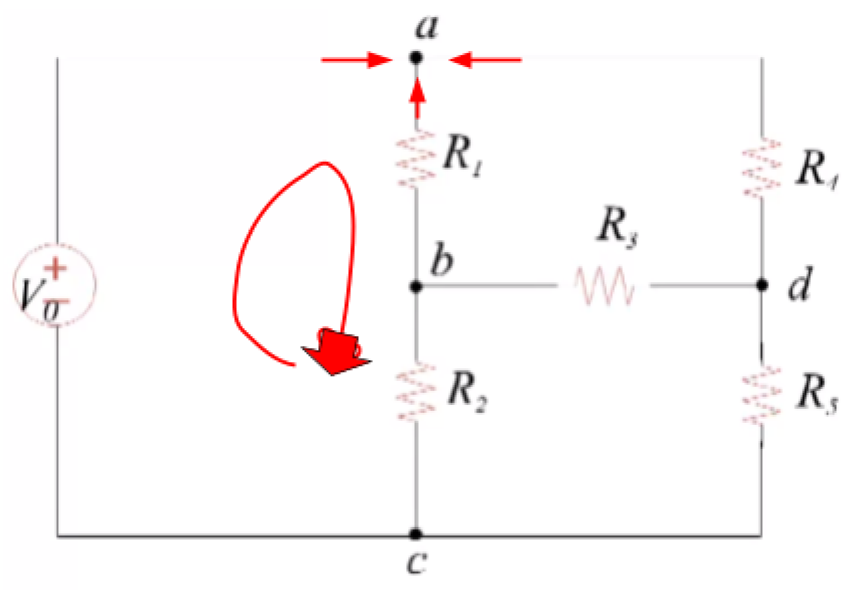
> Connecting ideal wires lumped elements that obey LMD to form an assembly results in lumped circuit abstraction.

Circuit Analysis

Kirchhoff's voltage law : the sum of the voltages at any loop is equal to 0

Kirchhoff's current law : the currents coming into a node is equal to 0

Note that Maxwell's equations can further simplified to obtain Kirchhoff's current and voltage laws.

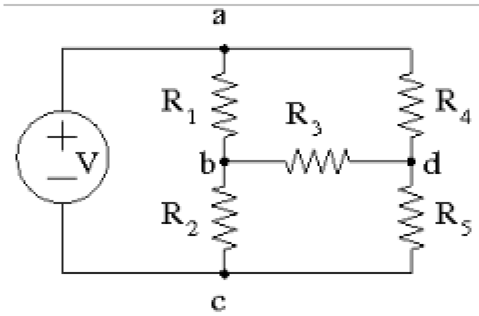


If we look at node A the current coming into the node, shown by the arrows, is equal to 0 according to Kirchhoff's current law.

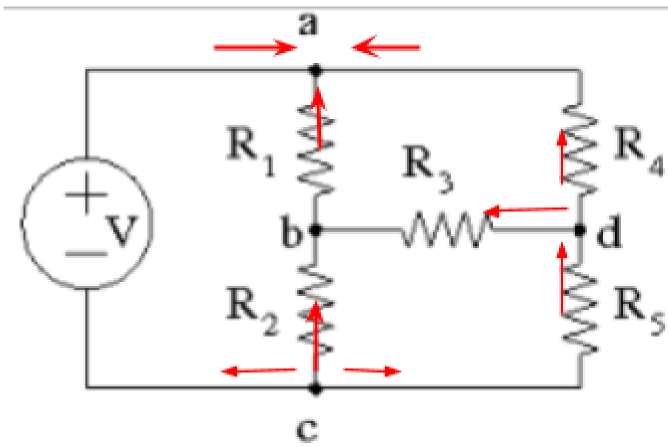
$$i(ca) + i(ba) + i(da) = 0$$

If we consider the loop, shown by the curved arrow, we can say that the voltage in a loop is equal to 0 according to Kirchhoff's voltage law.

$$v(ca) + v(ab) + v(bc) = 0$$



There are 4 nodes in the diagram.
How many Kirchhoff's current law (KCL) equations are independent?
The answer would be 3. It is important to note that there is one more node than there are independent KCL equations.
For example, if we have n nodes, there are $n - 1$ independent KCL equations.



When two nodes share a branch, the current entering one node from that branch is the negative of the current entering the other node from that branch. When two nodes share a branch, the current flowing between them cancels out because it's positive for one node and negative for the other.

Therefore, the sum of currents entering all nodes except one equals the current entering the remaining node. This means the KCL equation for the last node can be derived from the KCL equations of the other nodes.

We can also see that there are 7 loops in the above circuit :

a d b a,

b d c b,

a b c a,

a d c b a,

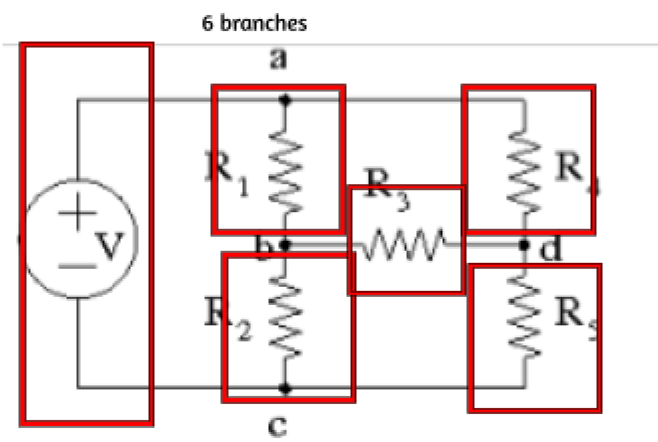
a d b c a,

a b d c a,

a d c a.

and there are 3 independent Kirchhoff's voltage law (KVL) equations:

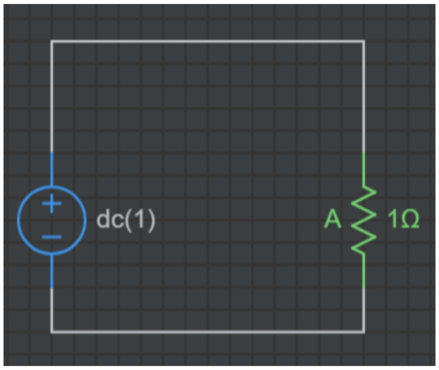
The number of independent KVL equations is generally $b - (n - 1)$, where b is the number of branches, and n is the number of nodes.



$6 - (4 - 1) = 3$

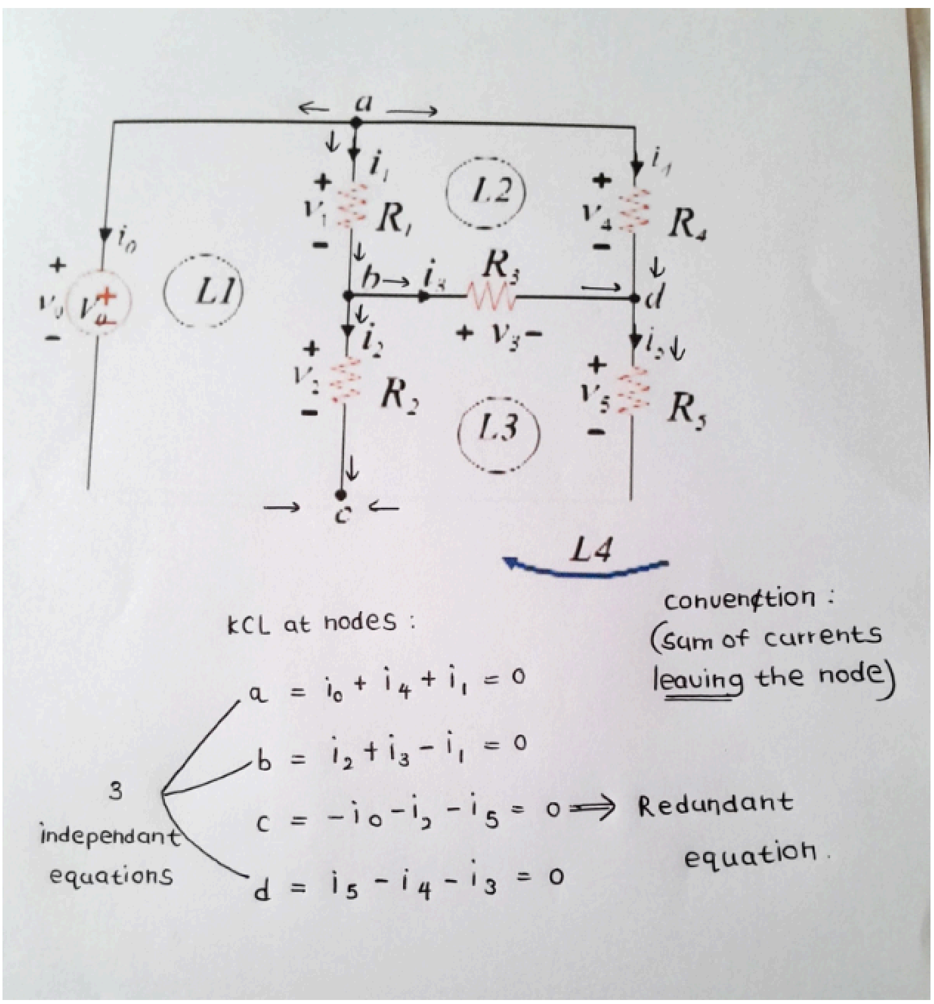
However, note that there are only three independent KVL equations. So, even if you write additional KVL equations for the fourth, fifth, sixth or seventh loops, they will result only in redundant equations.

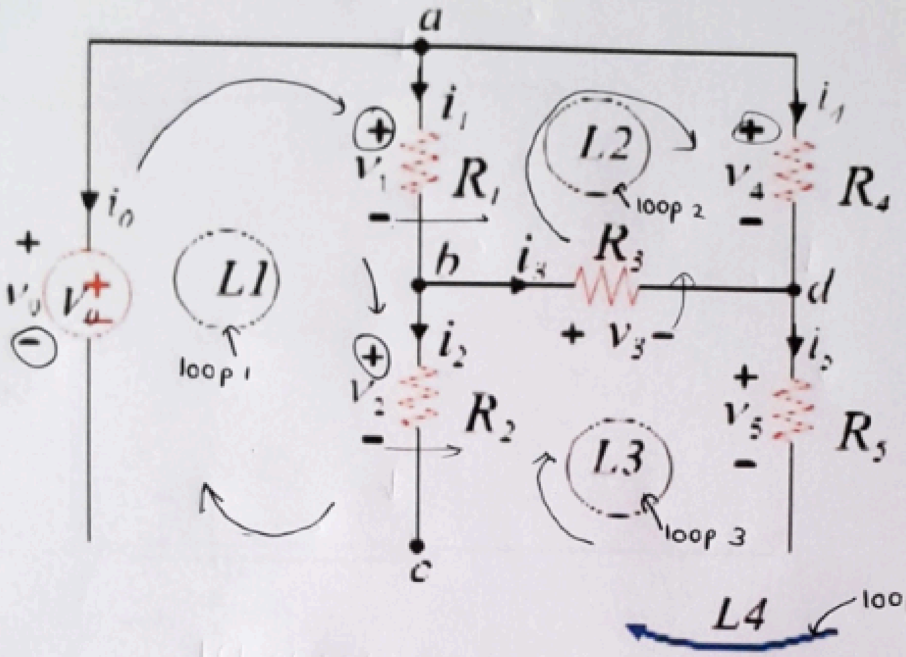
Moving to the KVL and KCL methods, we will look at an important convection called the Associated variables discipline.



If we take the current going into the positive voltage terminal then the power consumed by element A would also be positive.

Example of applying KVL and KCL and the v-i relationship for a circuit:





KVL for loops :

Convention : as we go around the loop, assign the <u>first</u> encountered sign to each voltage.

3 independant equations.

$L_1 = -v_0 + v_1 + v_2 = 0$

$L_2 = -v_1 + v_4 - v_2 = 0$

$L_3 = -v_2 + v_3 + v_5 = 0$

* 3 loops are enought to obtain 3 independant equations.

$L_4 = -v_0 + v_4 + v_5 = 0$ ⇒ redundant equation.

- - - - - - - - - - - -

Element v, i relationships :

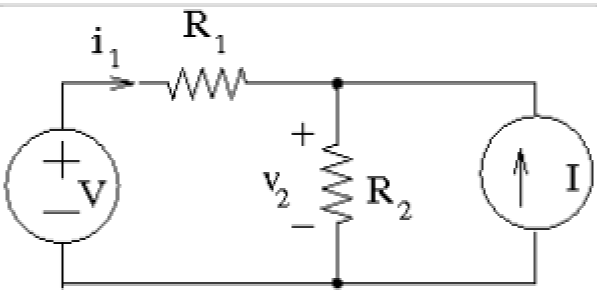
- $v_0 = V_0$
- $v_1 = i_1 R_1$
- $v_2 = i_2 R_2$
- $v_3 = i_3 R_3$
- $v_4 = i_4 R_4$
- $v_5 = i_5 R_5$

In a circuit, **V** typically represents the voltage source, which is the fixed voltage provided by a power source like a battery or a generator. **v** often represents the voltage drop or potential difference across a particular component in the circuit.

For calculations:

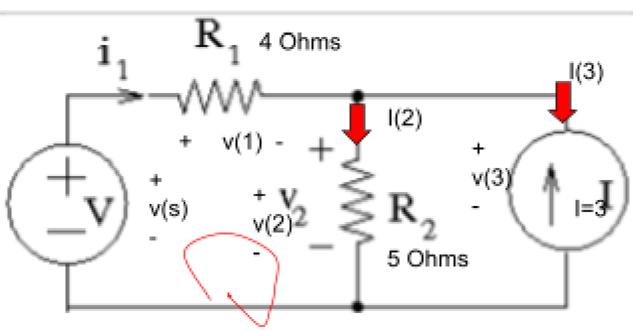
- Use **V** when dealing with the voltage provided by the source.
- Use **v** when calculating or analyzing the voltage across specific components in the circuit.

Trivia Question :



a) Calculate the voltage (V), v_2 across the resistor with resistance R_2
b) Calculate the power (W) dissipated by the resistor with resistance R_2
c) Calculate the current (A) i_1 through the resistor with resistance R_1
d) Calculate the power (W) dissipated by the resistor with resistance R_1
e) What is the power (W) supplied by the voltage source?
f) What is the power (W) supplied by the current source?

Note: The sum of the power supplied by the sources is the sum of the power dissipated by the resistors.



a) $v(s) = V = 2$

$v1 = i1 \times R1$

$v2 = i2 \times R2$

$i3 = \ - 3$

KVL = $- v(s) + v1 + v2 = 0$

KCL = $i2 - i1 + i3 = 0$

$i2 = i1 - i3$

$v2 = (i1 - i3) \times 5$

$v2 = (i1 - (- 3)) \times 5$

$i1 = v1/R1$

$v2 = ((v1/R1) + 3) \times 5$

$v2/5 = (v1/4) + 3$

KVL = $- v(s) + v1 + v2 = 0$

$v1 = v(s) - v2$

$$\frac{v2}{5} = \frac{v(s)-v2}{4} + 3$$

$$\frac{v2}{5} = \frac{v(s)-v2+12}{4}$$

$$\frac{4v2}{5} = 2 - v2 + 12$$

$4v2 = 10 - 5v2 + 60$

$9v2 = 10 + 60$

$9v2 = 70$

$v2 = \frac{70}{9} = 7.777777778V = 7.78V$

b) $P2 = v2 \times i2$

$v2 = i2 \times R2$

$P2 = 7.78V \times (7.78/5) = 12W$

c) $i2 = i1 - i3$

$i1 = i2 + i3 =(7.78/5) - 3 = - 1.44A$

d) $P1 = v1 \times i1$

$v1 = i1 \times R1$

$P1 = - 1.44 \times 4 \times - 1.44 = 8.3W$

e) $PV = vV \times i1$

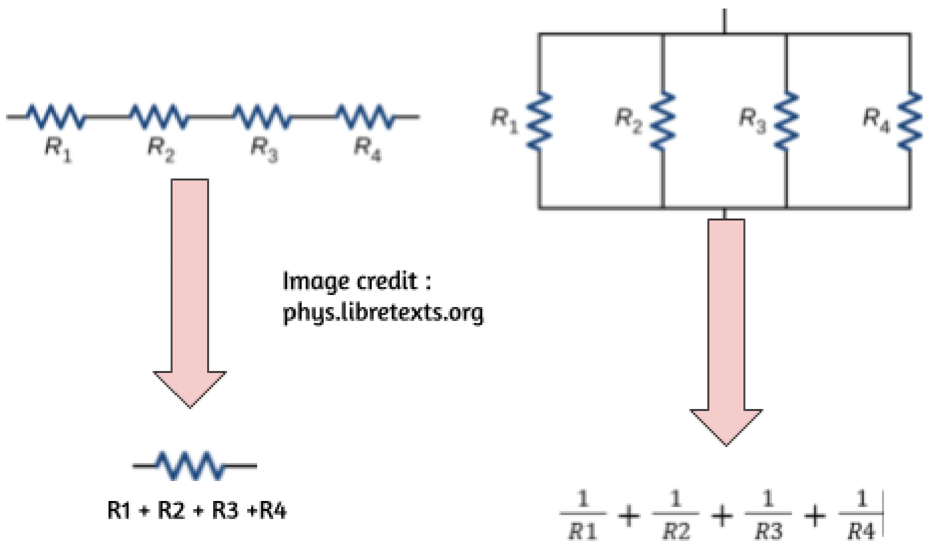
$P1 = 2 \times - 1.44 = - 2.88W$

f) $PI = vI \times I$

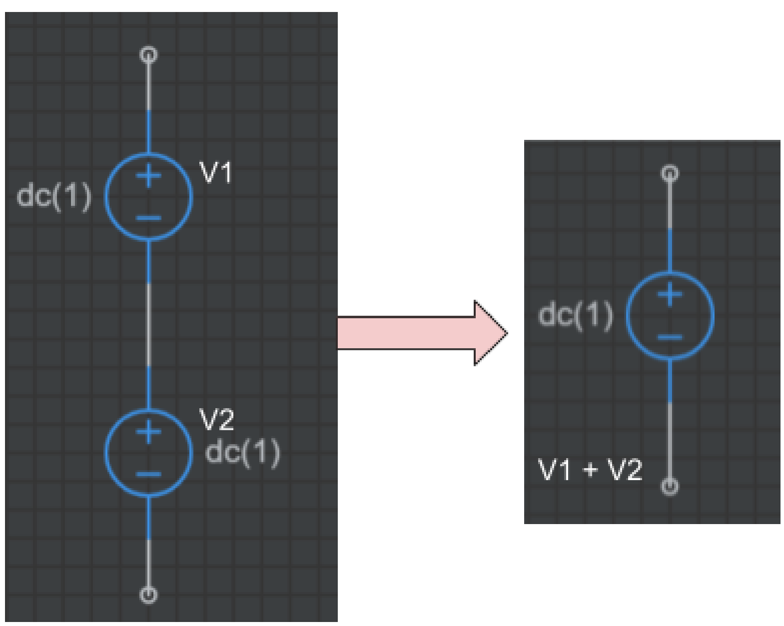
$PI = 7.78 \times 3 = 23.34 = 23W$ (The potential difference in the current source is equal to $v2$)

Element combination Rules

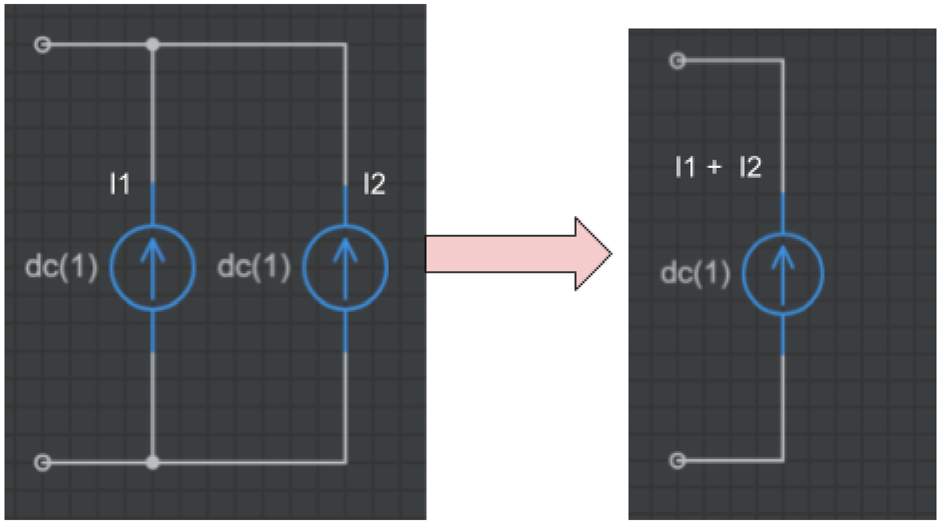
1) If we have resistors connected in parallel and in series we can further simplify them as shown below :



2) Two or more voltage sources simplify as shown below :



3) Two or more current sources simplify to :



Node analysis

Steps :

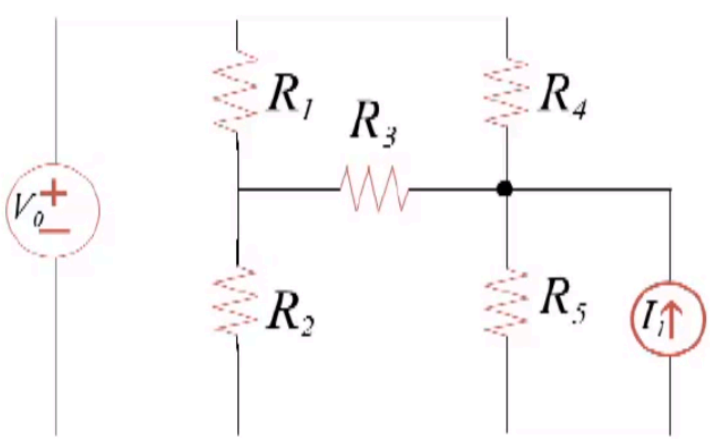
1) Select the reference node (ground) from which voltages are measured.

2) Label voltages of other nodes relative to the ground (known as primary unknowns)

3) Write KCL or KVL or substitute device laws for all except the ground node.

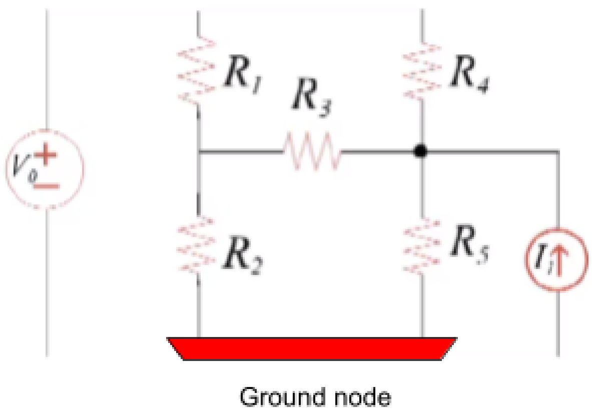
4) Solve the node voltages

5) Back solve for branch voltages and currents (secondary unknowns)

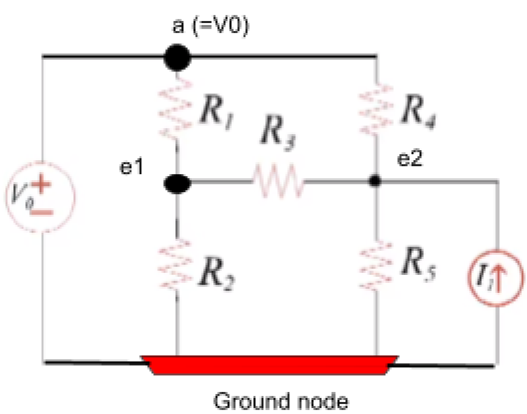


When considering the reference node, we must pick the node with the largest number of edges coming into it and a node with many sources connected to it.

Step 1:

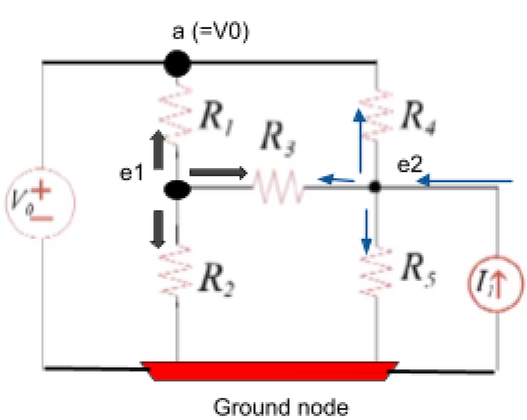


Step 2:



'a' is V0 because it is connected to the ground by the voltage source.

Step 3:



Conductance is $Gi = \frac{1}{R1}$ (for convenience)

Convection for KCL : sum of the current leaving the node (=0)

$KCL\ at\ e1$:

$$\frac{e_1 - v_0}{R_1} + \frac{e_1 - e_2}{R_3} + \frac{e_1}{R_2}$$

$$[(e1 - V0) \times G1] + [(e1 - e2) \times G3] + [(e1) \times G2] = 0$$

$KCL\ at\ e2$:

$$\frac{e_2 - e_1}{R_3} + \frac{e_2 - V_0}{R_4} + \frac{e_2}{R_5} - I_1$$

$$[(e2 - e1) \times G3] + [(e2 - V0) \times G4] + [(e2) \times G5] - I1 = 0$$

Step 4 (solve):

$$[(e1 - V0) \times G1] + [(e1 - e2) \times G3] + [(e1) \times G2] = 0$$

$$e1(G1 + G3 + G2) + e2(-G3) = V0 \times G1$$

$$[(e2 - e1) \times G3] + [(e2 - V0) \times G4] + [(e2) \times G5] - I1 = 0$$

$$e1(-G3) + e2(G3 + G4 + G5) = V0 \times G4 + I1$$

Now we have to solve the two equations shown in red and pink using simultaneous equations.

This in matrix form :

$$\begin{bmatrix} G_1+G_2+G_3 & -G_3 \\ -G_3 & G_3+G_4+G_5 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}$$

conductivity matrix — unknown node voltages — sources

Solve

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \frac{\begin{bmatrix} G_3+G_4+G_5 & G_3 \\ G_3 & G_1+G_2+G_3 \end{bmatrix} \begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}}{(G_1+G_2+G_3)(G_3+G_4+G_5)-G_3^2}$$

$$e_1 = \frac{(G_3+G_4+G_5)(G_1V_0)+(G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3^2+G_3G_4+G_3G_5}$$

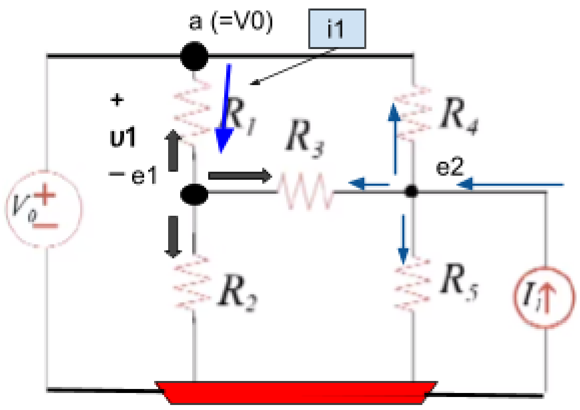
$$e_2 = \frac{(G_3)(G_1V_0)+(G_1+G_2+G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3^2+G_3G_4+G_3G_5}$$

(same denominator)

(We can use computer systems to solve)

*Image credit: MIT 6.002x

Step 5:

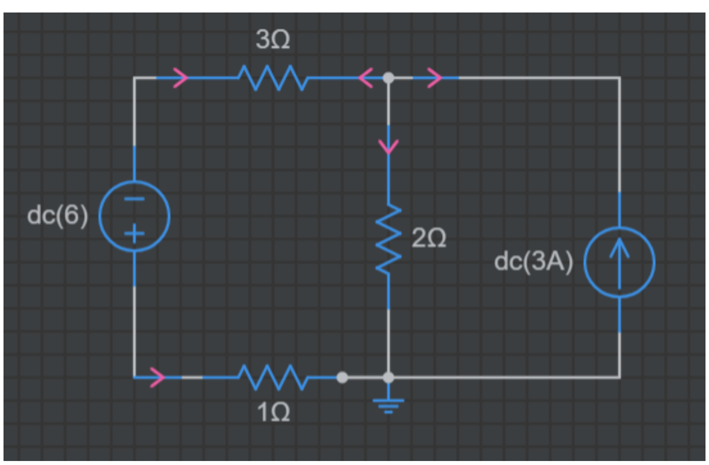


$$v1 = V0 - e1$$

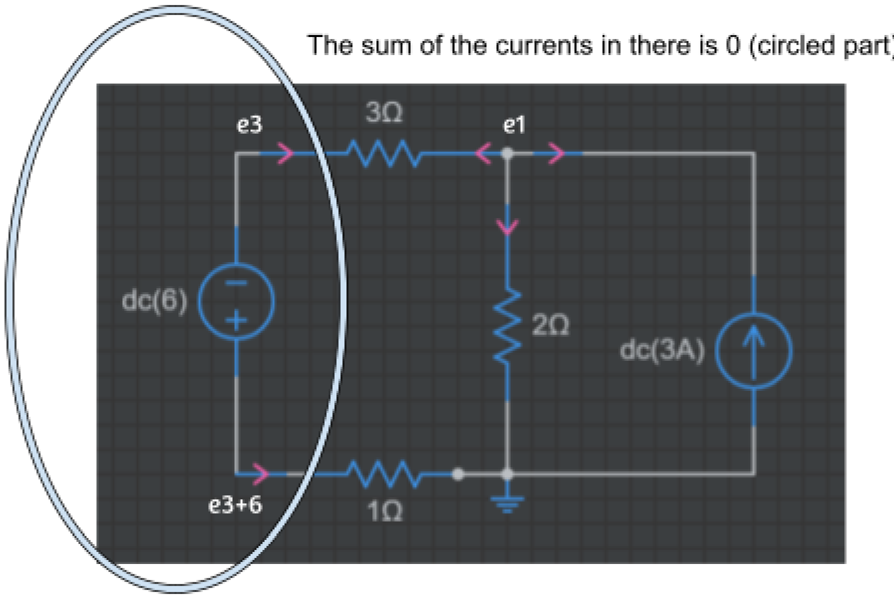
$$i1 = \frac{v1}{R1} = \frac{(V0 - e1)}{R1}$$

Similarly, we can find the rest of the voltages and currents.

<u>**Nodal analysis with Floating Voltage source.**</u>



The position of the reference node is changed. The voltage source is not directly connected to the reference node.

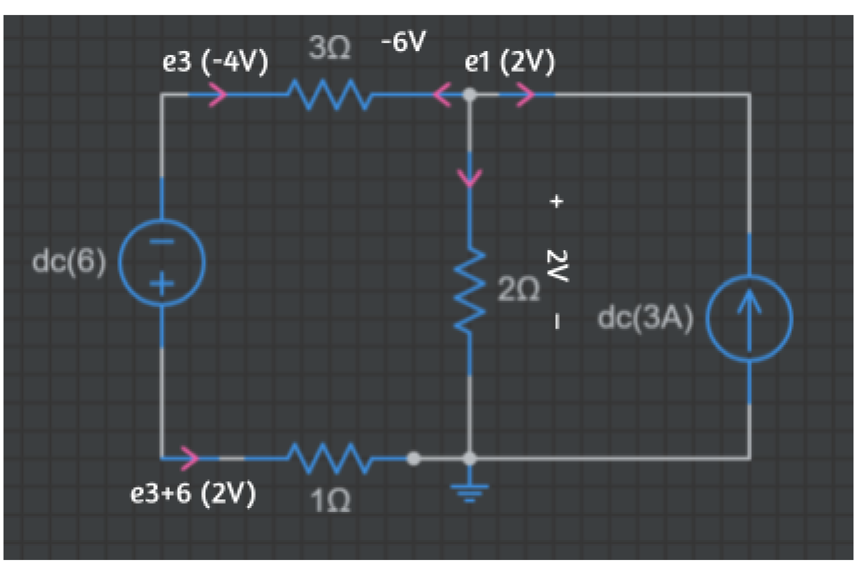


$$\frac{e1-e3}{3} + \frac{e1-0}{2} - 3 = 0$$

$$\frac{e3-e1}{3} + \frac{e3+6}{1} = 0$$

Solving for $e1$ *and* $e3$:

$e1 = 2$ *and* $e3 = -4$



Voltage Divider Rule

The voltage divider rule is a simple way to determine the voltage across a particular component in a series circuit.

The condition for using this equation is that the current through both resisters should be the same.

If we have two resistors, $R1 \ and \ R2$:

The voltage across $R1$:

$$V1 = \frac{R1}{R1+R2} \times V(in)$$

The voltage across $R2$:

$$V2 = \frac{R2}{R1+R2} \times V(in)$$

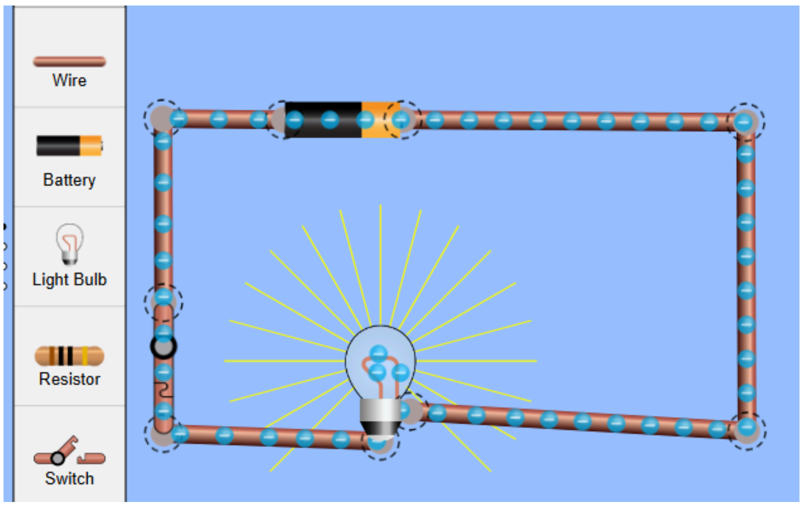
Practical Applications:

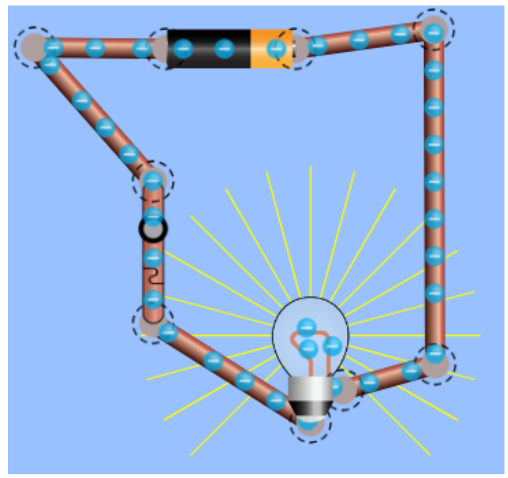
1) Adjusting Signal Levels: Voltage dividers are used to create reference voltages or reduce the magnitude of a voltage to a desired level.

2) Sensor Readings: Often used in analog sensors to scale down the output voltage.

Circuit Abstraction

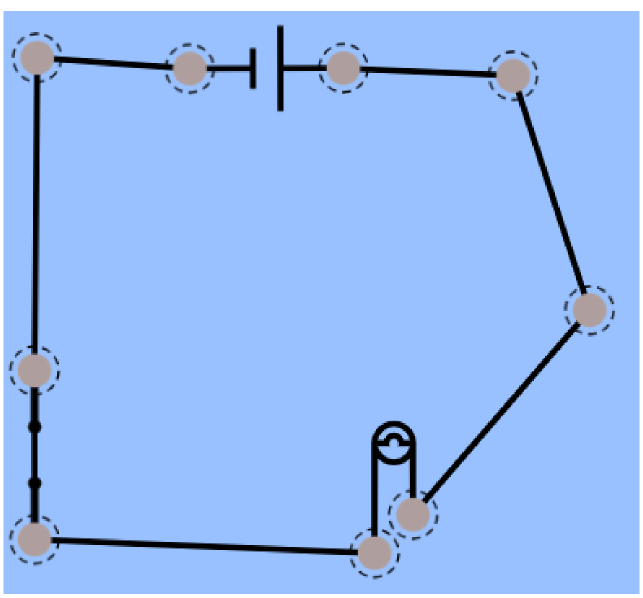
If a light bulb is connected to a battery with a connecting wire and the switch is closed, the light bulb would light up. If we change the shape of the wires by tangling them, the light bulb would still light up showing us that the geometry of the wires does not affect the circuit. The important point is that the connections should be made correctly.





The bulb lights even when the geometry of the connecting wires are altered.

(Phet simulations)

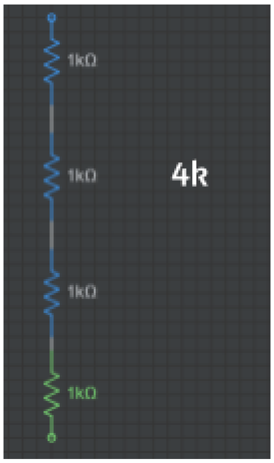


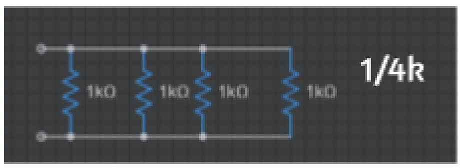
(we use a circuit diagram like this (also known as a schematic diagram) to simplify the circuit to understand easily)

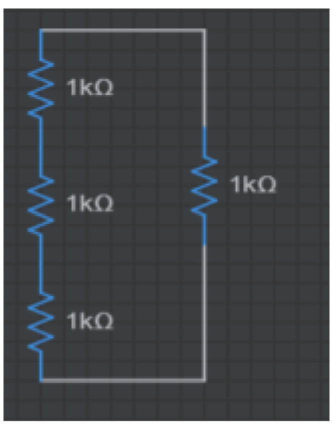
Trivia Question 1.

Question: Using four, 1k resistors, produce a resistor of 3/5k and a resistor of 5/3k.

Let's look at the possible ways of connecting these resistors :



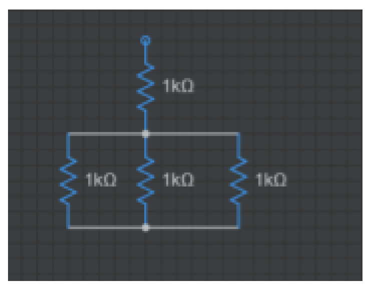




1k+1k+1k=3k

1/R = 1/3k + 1/1k = 4/3 k

R = ¾ k

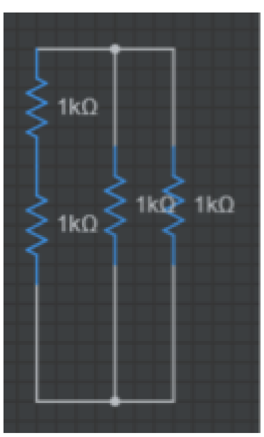


1/R = 1/1k + 1/1k + 1/1k = 3 k

R = 1/3k

R = 1k + 1/3k = 4/3k

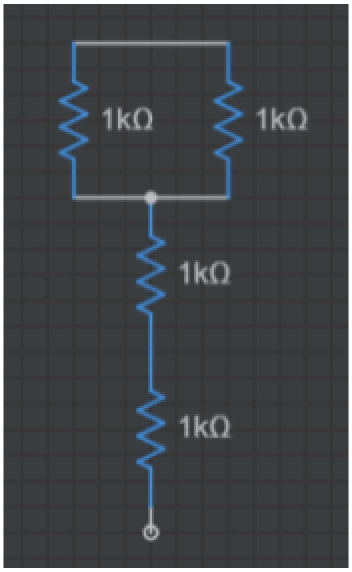
R = 4/3 k



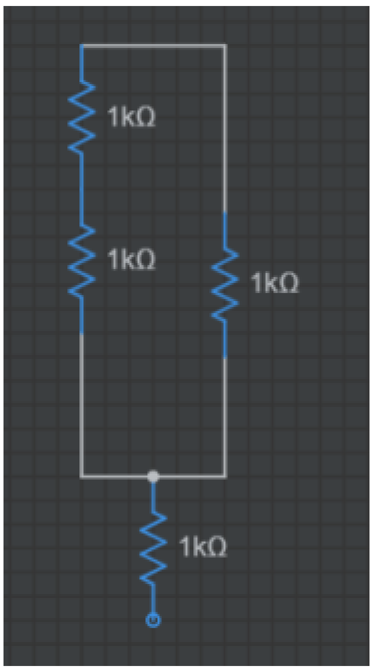
1k+1k+=2k

1/R = 1/2k + 1/1k + 1/1k = 5/2 k

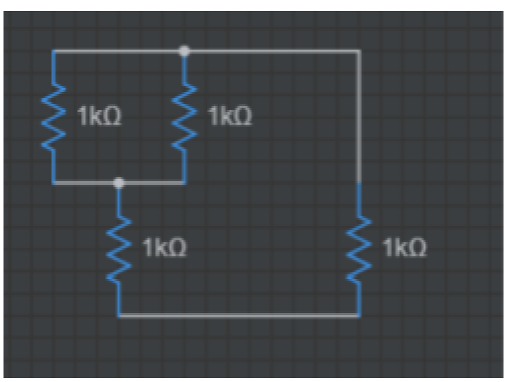
R = ⅖ k



R = 5/2 k



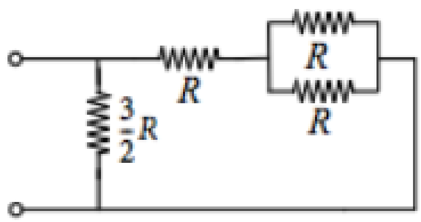
R = 5/3 k



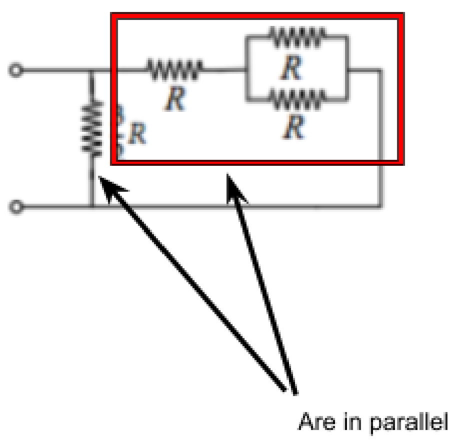
R = 3/5 k

Trivia Question 2.

Question: Find the resistance of the circuit below :



Answer :



$$\frac{1}{R} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} ---> \ R = \frac{R}{2}$$

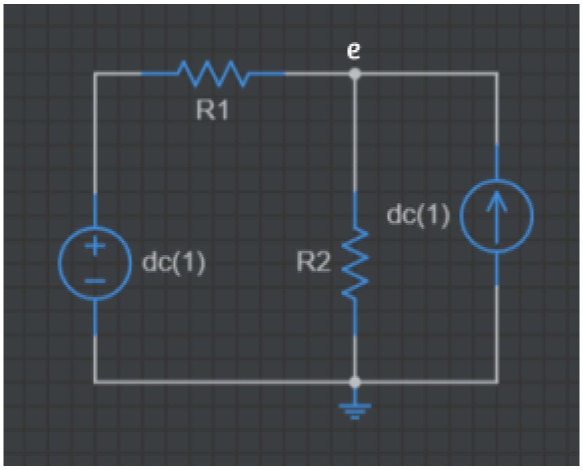
$$R + \frac{R}{2} = \frac{3}{2}R$$

Resistance of the whole thing :

$$\frac{1}{R} = \frac{2}{3R} + \frac{2}{3R} = \frac{4}{3R} ---> \ R = \frac{3}{4}R$$

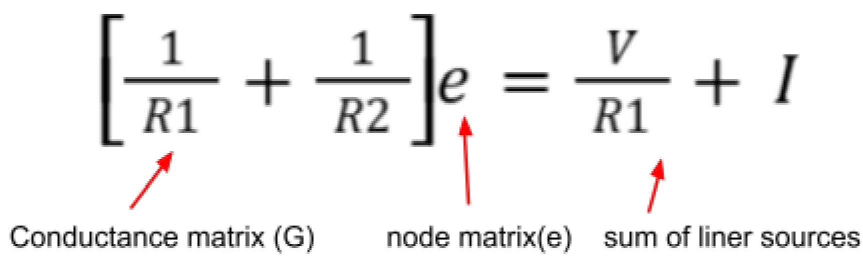
Linearity

Linear circuits have linear relationships between voltages and currents.



$$\frac{e-V}{R1} + \frac{e}{R2} - I = 0$$

$$\left[\frac{1}{R1} + \frac{1}{R2}\right]e = \frac{V}{R1} + I$$



$$\frac{R1+R2}{R1R2} \times e = \frac{V}{R1} + I$$

$$e = \frac{V}{R1} + I \times \frac{R1R2}{R1+R2}$$

$$e = \frac{VR2}{R1+R2} + \frac{R1R2I}{R1+R2}$$

So e is in the form of :

$$e = a1v1 + a2v2 + \ldots + b1i1 + b2i2 + \ldots \quad , where\ a\ and\ b\ are\ constants.$$

Properties of Linearity

1) Homogeneity :

If you multiply the input by a constant factor, the output will be multiplied by the same constant factor.

Here is a system below :

If you multiply by a constant "a" :

Thus, if we multiply the inputs by a constant, the output will also be multiplied by a constant according to homogeneity.

2) Superposition :

The total response of a linear system to multiple inputs is the sum of the responses to each individual input applied separately.

If we have a system shown below :

What will be out output?

With the method of superposition, we can use a divide-and-conquer technique.

We can set the V2 to be 0 and apply just V1:

Then we can set V1 to be 0 and apply just V2:

Now if we combine these, the output would be the sum of the individual outputs with each one of those applied alone.

The theory of superposition says that if we take corresponding sets of inputs and sum them up, then the output would be a sum of the two partial outputs.

Now let's look at the use of the superposition method in circuit analysis:

This works for linear circuits with independent sources.

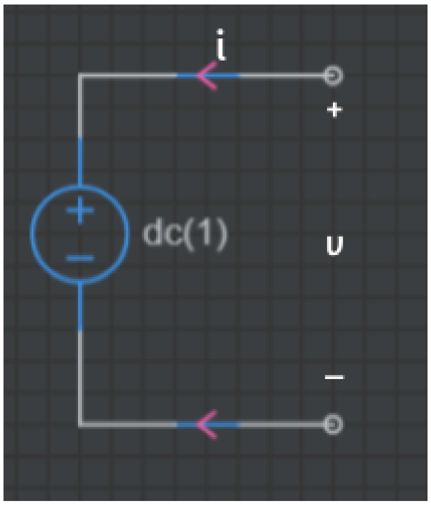
Steps :

1) Find the response of the circuit to each source acting alone. (we turn off all the sources and find the response to each source acting alone)

2) Sum the individual responses.

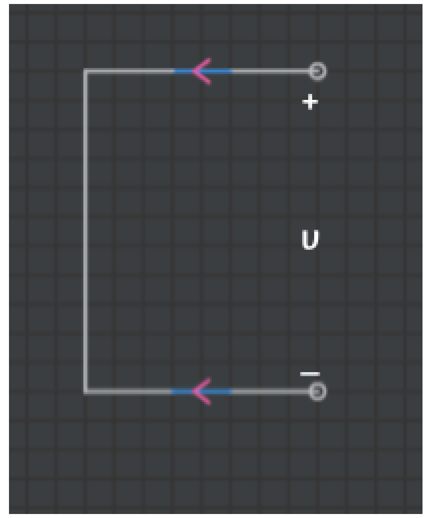
Meaning of each source acting alone :

To set a voltage to 0, you short it out and to set a current to 0, you open-circuit it.

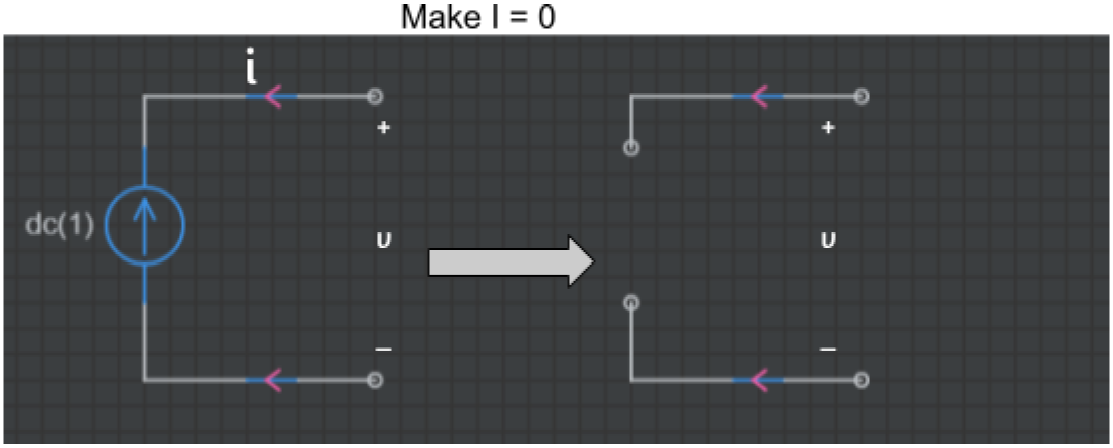


(make V = 0,short the circuit)

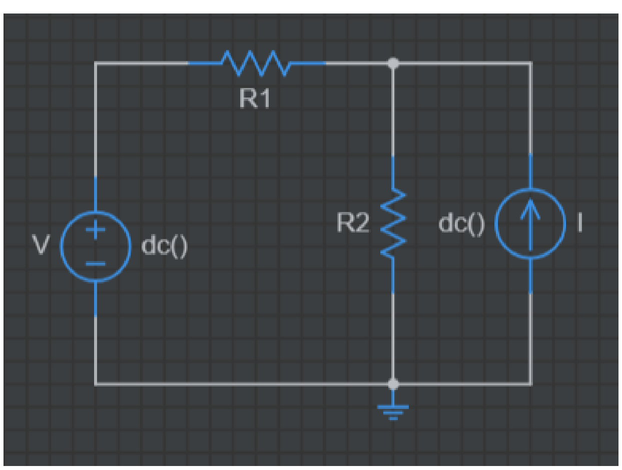
—------------------------->



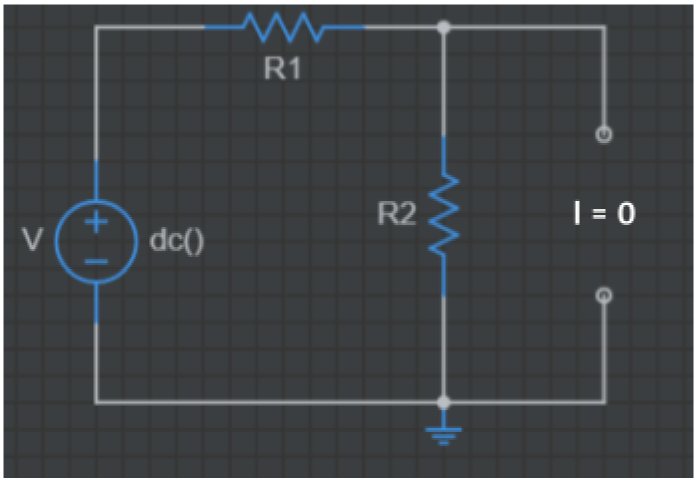
Make I = 0



Example :



ev



By making it an open circuit, we can make I = 0 , so the voltage source is acting alone.

The node voltage would be 'e', so that 'e' would become the partial response of the circuit to the voltage acting alone.

$$Applying\ KCL\ we\ have\ ; \ - V + R1i + R2i = 0$$

$$voltage\ across\ R2\ is\ ev - 0 = R2i$$

$$so\ i = \frac{ev}{R2}$$

$$- V + R1 \times \frac{ev}{R2} + ev = 0$$

$$\frac{R1ev}{R2} + ev = V$$

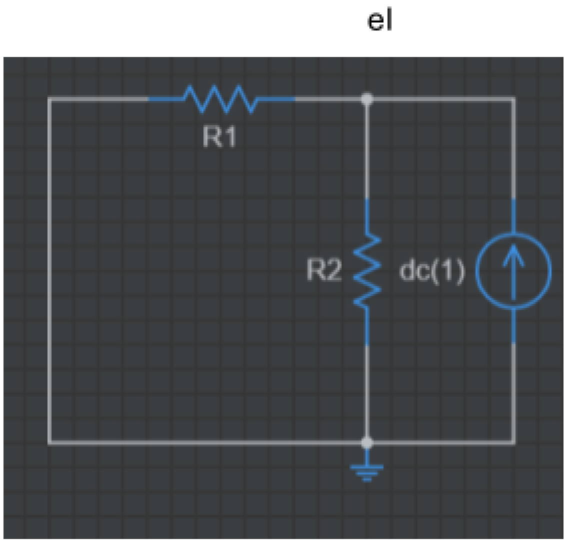
$$\frac{R1ev}{R2} + \frac{R2ev}{R2} = V$$

$$\frac{ev(R1+R2)}{R2} = V$$

$$ev = V \times \frac{R2}{(R1+R2)}$$

Next, to get a response for the current acting alone, we need to build a circuit with the voltage source turned off.

To do so, we are making a short circuit as shown below where the V = 0:



We need to find the voltage eI, which is a result of the current source acting alone.

eI = resistance time the current

The resistance is ;

$$eI = \frac{1}{R1} + \frac{1}{R2} = \frac{R1R2}{(R1+R2)} \times I$$

Step 2:

Sum the 2 partial voltages :

$$e = eV + eI = \frac{R1R2}{(R1+R2)}I + \frac{R2}{(R1+R2)}V$$

Thus, this would be the answer according to the superposition method.

Thevenin Method

Say you have a network with a lot of independent voltage and current sources and resistors and what we want to find is the voltage v, where a current i flows.

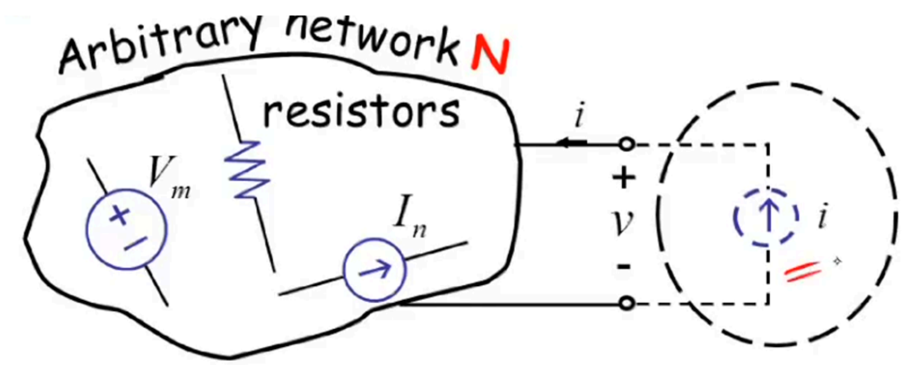
In this case we can use superposition to help solve the matter.

An external excitation is applied and that is current i.

By superposition, voltage v will be some combination of the sources in the circuit.

Thevenin's Theorem states that any linear circuit with multiple sources and resistors can be reduced to a single voltage source (V_{TH}) and a single series resistor (R_{TH}).

resistors ($R1, R2, R3$,.... Rn), independent voltage sources ($V1, V2, V3$,.... Vm), and independent current sources ($I1, I2, I3$,.... In)



An external excitation is created due to current i. We have to determine v.

In this case, when you want to figure out the current or the voltage at some point in the circuit, we can use the method of superposition.

We want to figure out the contribution to the voltage v due to the voltage source Vm.

To do that, we have to set all the currents including i to 0 and all the voltages $V1$ to $(Vm - 1)$ to 0 so that it allows us to look at the contribution on v due to Vm acting alone.

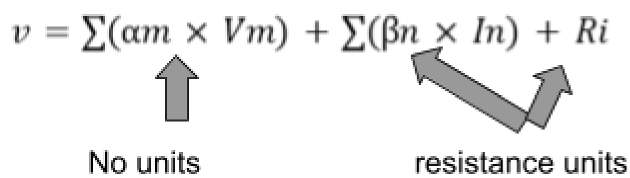
When these are turned off we get a huge resistor network giving us a multiplier αm in this case.

Similarly, we can do this to all the voltages $V1,\ V2\ etc.$. and then we can sum them up.

Moving to currents, we need to set all the currents from $I1\ to\ (In - 1)$ to 0, i to 0 and the $Vm's$ to 0 and this time we get some factor βn. Similarly, we can have $I1, I2\ etc.$. acting alone and finally sum them up.

For i to act alone, we need to set all the current and voltage sources to 0. Then we will get the i and some multiplier which is the resistance of the big resistor network. (external excitation acting alone)

$$v = \sum(\alpha m \times Vm) + \sum(\beta n \times In) + Ri$$

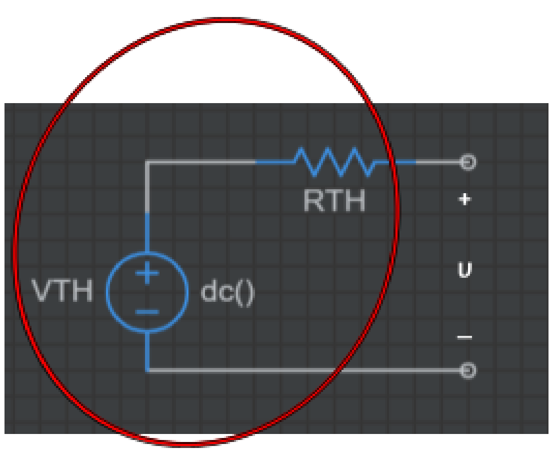


The $Vm\ and\ In$ related terms are independent of the external excitation and behave like a voltage. Thus we can call it V_{TH}.

Ri has 'i' showing a current and the multiplier R is independent of the external excitation and behaves like a resistor. Thus we can call it R_{TH}.

$$new\ equation\ for\ v:\ v = V_{TH} + R_{TH}i$$

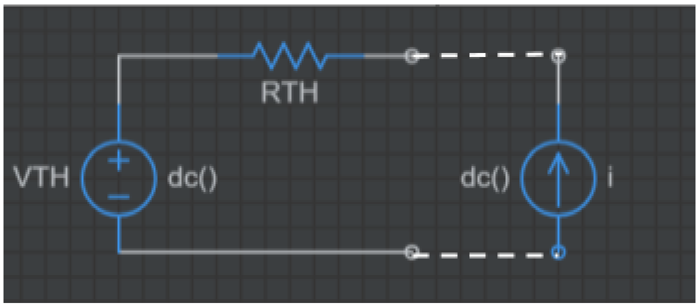
Thus we get a simplified circuit as shown below :



Network N is simplified as shown below

This network is called the Thevenin equivalent network.

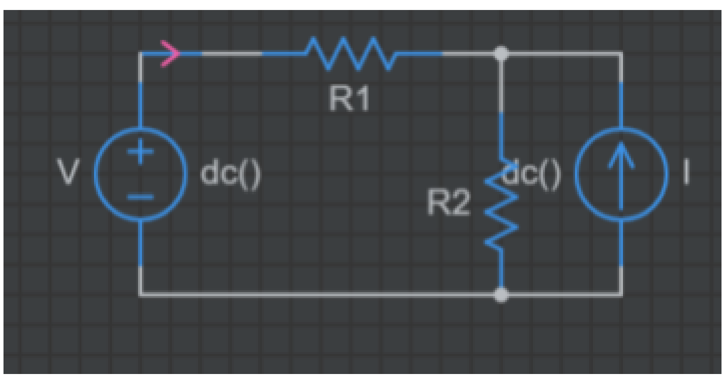
Then we can connect the external excitation, ‘i’, to compute the voltage, v across the terminals.



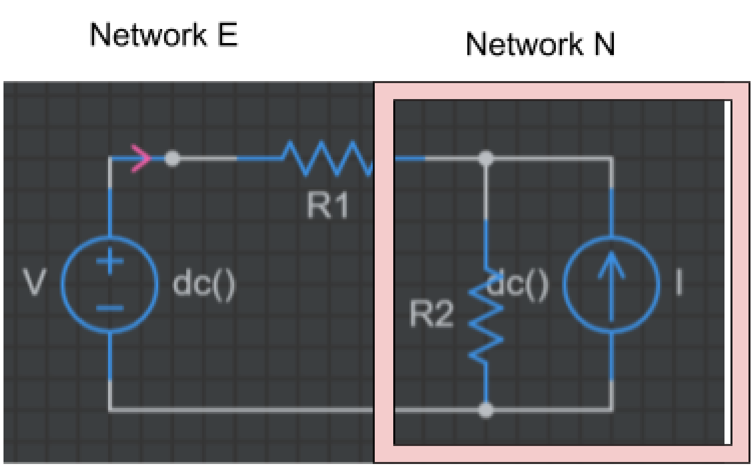
V_{TH} —----------> Open circuit voltage seen at the terminal pair. (port)

R_{TH} —----------> Resistance of network seen from the port. ($Vm's,\ In's\ set\ to\ 0$)

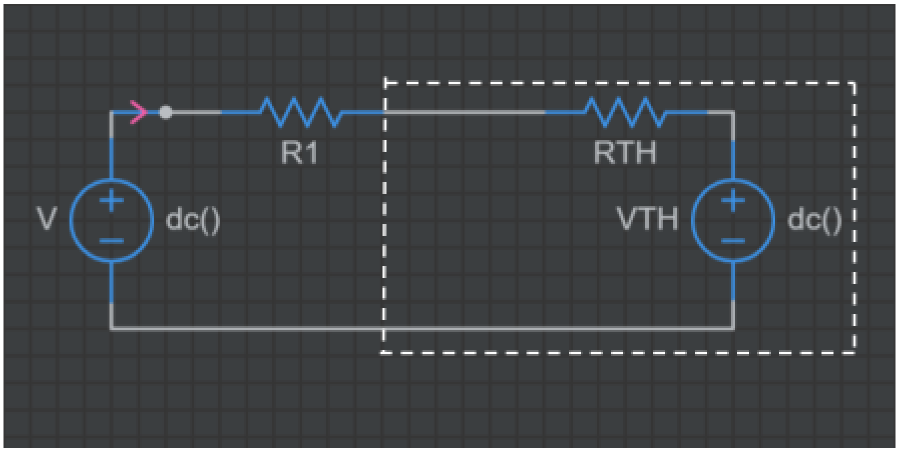
Example : Find $i1$



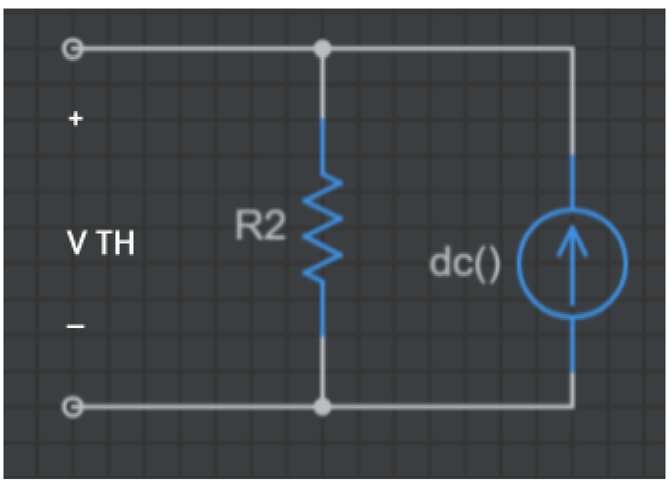
According to the Thevenin method, the first step is to replace network N (arbitrary network) with a Thevenin equivalent.



Which looks like this :



We need to find V_{TH} and R_{TH}.

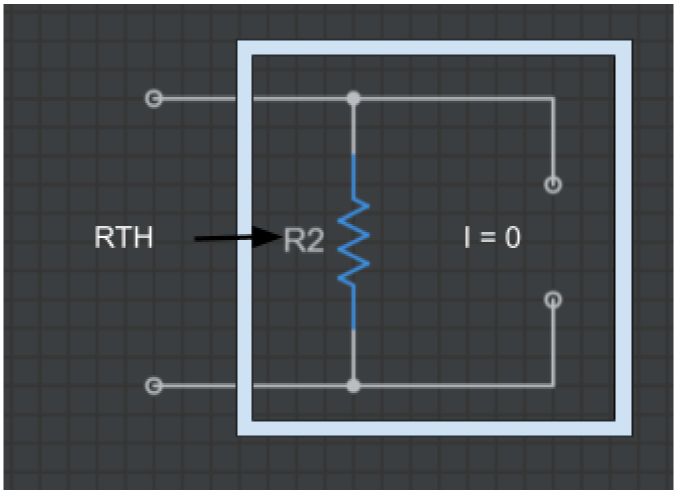


The open circuit voltage is the V_{TH} (Thevenin Voltage)

$$V_{TH} = I \times R2$$

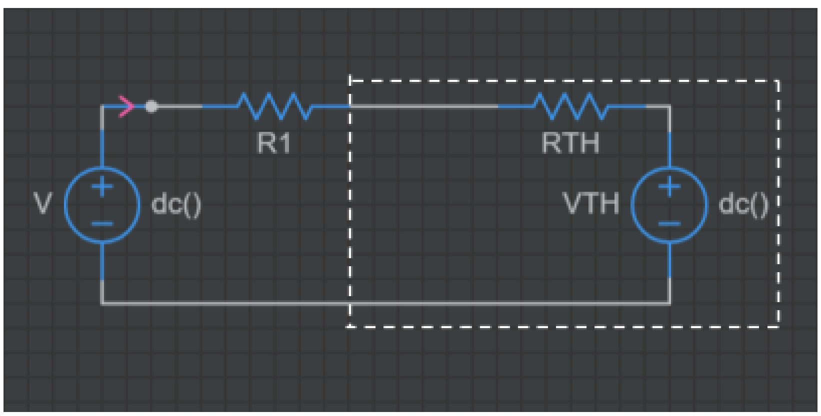
Then we have to find R_{TH} and we need to do so by turning off all the independent sources and measuring the resistance.

In order to make I = 0 we need to make an open circuit.



Thus R_{TH} will be equal to $R2$.

Step 2 : Solve with external network E



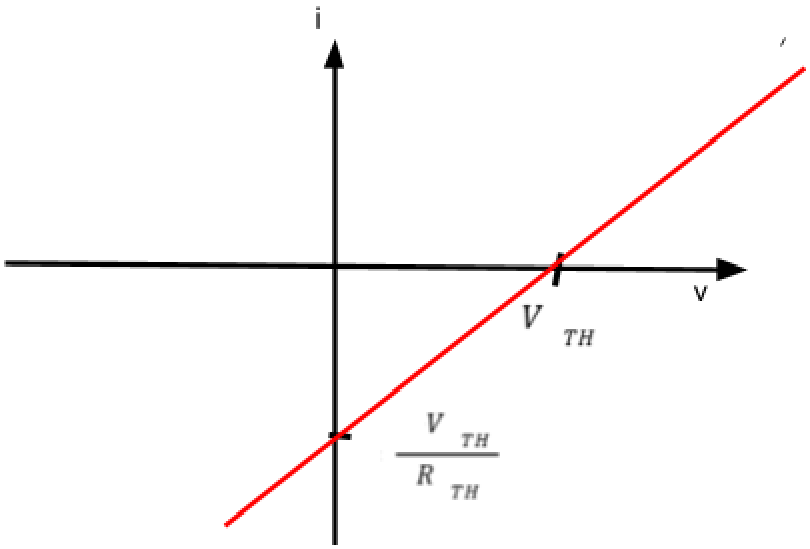
** $i1 = \frac{V - V_{TH}}{R1 + R_{TH}}$

Now let's look at the $v - i$ relationship graphically.

$$v = V_{TH} + R_{TH} i$$

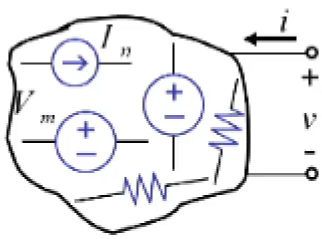
We need to draw a pair of axes with 'i' in the y-axis and 'v' in the x-axis.

Open circuit: i=0	$v = V_{TH}$
Short circuit: $v = 0$	$0 = V_{TH} + R_{TH}i$ $-V_{TH} = R_{TH}i$ $i = \frac{-V_{TH}}{R_{TH}}$

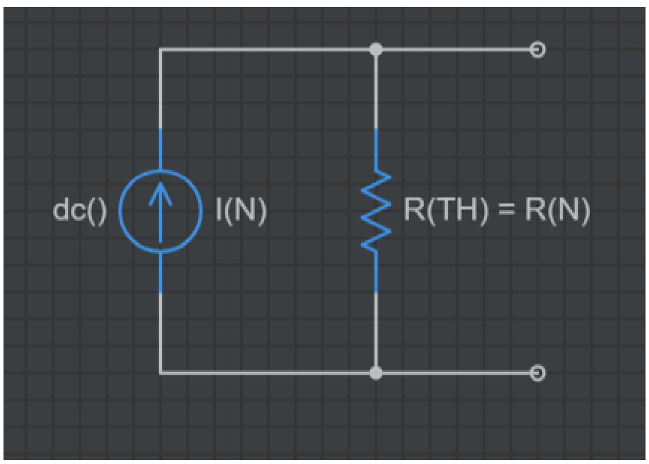


The slope of the line is $\frac{1}{R_{TH}}$

Norton Method

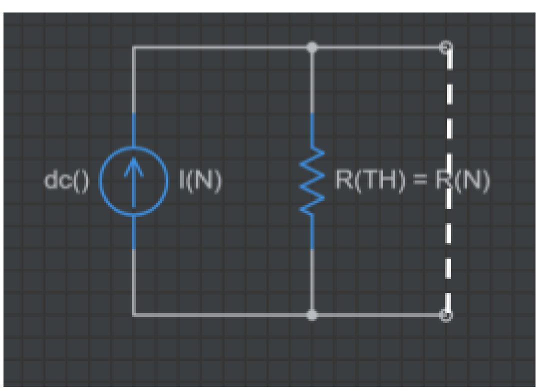


We can draw the Norton equivalent :



The Norton equivalent has a current source and resistors parallel to it.

So to obtain the current $I(N)$, we need to measure the short circuit current seen at the port.



Then we measure the current I. The current is $I(N)$ which is also known as the Norton current.

The Norton resistance $R(N)$ is equivalent to $R(TH)$. We need to turn off all the independent voltage sources and current sources and simply measure the resistance value at the port, without the short circuit.

$R(TH) \; = R(N) \; Thevenin\, resistance\, is\, equal\, to\, Norton\, resistance$

$$I(N) \; = \frac{V(TH)}{R(TH)}$$

We can calculate the Norton current by dividing the Thevenin voltage and the Thevenin resistance.

This shows the link between the Thevenin and Norton patterns.

Digital abstraction.

Analogue signals are easily affected by noise, which can distort the signal when it's sent through long wires. This makes it hard for the receiver to understand the original signal accurately, as even small amounts of noise can cause confusion and errors and difficulty differentiating voltage values of small differences.

Analogue signals simply lack noise immunity.

It is important to note that when connecting two circuits, you need to connect both the signal wire and the ground wire. The ground wire provides a return path for the current, ensuring that the circuit operates correctly. Connecting to the ground is necessary for the signal to be transmitted properly.

We need a way to discretize the values and that is to convert them to digital signals."Discretize" means converting a continuous signal or value into a series of distinct, separate values. For example, in digital signal processing, continuous analogue signals are often discretized into digital form, represented by a series of discrete numbers.

So we use two sets of values, high and low which are often referred to as '1' and '0'.

Digital signals are less affected by noise and interference, ensuring clearer and more accurate data transmission. Digital signals maintain their quality over long distances, unlike analogue signals which degrade.

It is important to note that digital circuits are non linear.

In digital systems, noise can be introduced into signals. By using 0 and 1, the receiver can still receive the correct signal even in noisy environments from the sender.

Thus, noise immunity is a key feature in digital systems.

Noise margin is the measure of how much noise a digital signal can tolerate before it is misinterpreted. High Noise Margin means that the signal can withstand a lot of noise without errors. Low Noise Margin means the signal is more vulnerable to noise and errors.

Static Discipline

In digital systems, establishing a convention between logical values and voltage levels ensures that signals are correctly transmitted and received.

For example, if a voltage below 2.5 volts is considered a logical 0 and a voltage above 2.5 volts is considered a logical 1, the sender can transmit signals within these ranges.

The receiver, using these thresholds, can accurately interpret the signals.

What if the voltage is about 2.5V?

For this reason, let's say a forbidden region in the middle where the senders and receivers cannot send or receive values could be created.

For example, we can define a region from 2 volts to 3 volts to be our forbidden region. Anything above 3 volts would be '1' and anything below 2 volts gives a '0'.

We can do so with impunity because it is our choice as to what discipline we agree on in our digital playground.

Does this work?

Unfortunately, this does not work. In digital systems, if signals are sent exactly at threshold voltages (e.g., 3 volts for high), any noise can push them into the "forbidden region," causing errors. This means there's no noise margin.

So what are we going to do?

We need to hold the sender to tougher standards.

We can tell the sender that '1' will be between V(OH) voltage output high to 5 volts for example and a logical low, '0', the sender needs to send signals between V(OL), voltage output low, and 0V. These are small ranges (the sender needs to send signals that are small in range).

For the receiver, it will be a little different.

The receiver needs to interpret the signals over a large range. So a logical high, '1', would be in a range of V(IH), voltage input high, to 5 volts. On the other hand, a '0', would be from V(IL), voltage input low, to 0 volt. These are large ranges.

And between V(IH) and V(IL) there is a forbidden region, which is a much smaller region, where a signal cannot be present.

Let's look at noise margins.

Noise margins for a logical high :

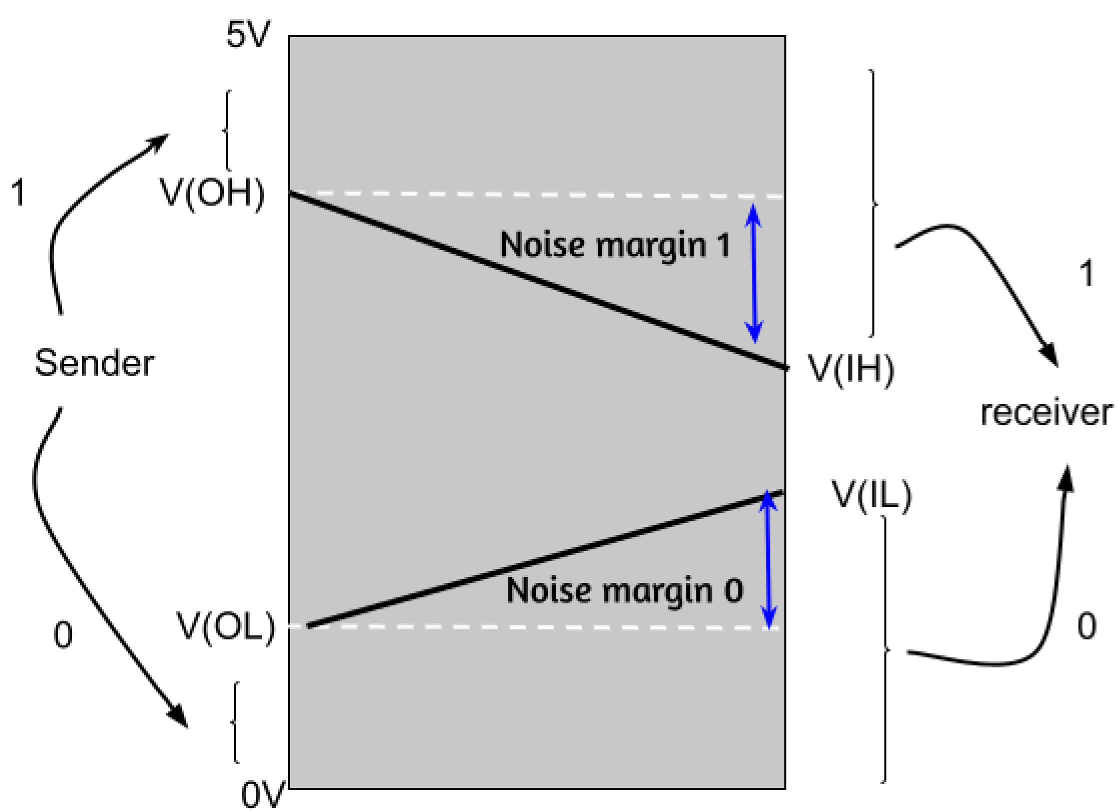
This means how much noise we can add to the system and still interpret the weakest possible 1 as a valid 1. In this case, the weakest one that the sender can send would be V(OH). If we add a lot of noise to the system, then the receiver will interpret the signals down to V(IH) as a logical one.

This means that the signal can be corrupted by this range but the receiver will still get the correct answer.

Therefore, the noise margin for a '1' would be between V(OH) and V(IH).

Similarly, the noise margin for a '0' would have been between V(IL) and V(OL).

V(OH), V(OL), V(IH), and V(IL) thresholds define a discipline or a standard that the digital devices follow so they talk to each other.



As long as we stick to this convention, we can all be working correctly and not making mistakes.

Digital systems follow static discipline. The static discipline is characterized by the four voltage thresholds, V(OH), V(IH), V(IL) and V(OL).

This discipline says that if the inputs to the system meet the valid input thresholds, then the output will meet valid output thresholds.

These thresholds make sure that signals are interpreted correctly despite noise interference.

Digital Logic Circuits

The digital signals can be represented by 0s and 1s.
Boolean logic refers to processing 0's and 1's (binary/boolean values)

Let's look at the AND gate :
If X and Y is true, then Z is true, else Z is false

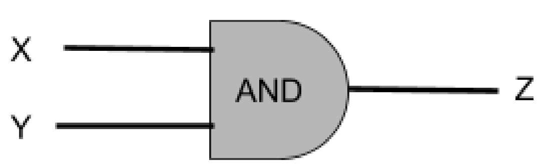
Z = X AND Y/ Z = X.Y

Possible combinations (Truth table Representation)

Z (X AND Y)	X	Y
0	1	0
0	0	1
1	1	1
0	0	0

Z = X.Y → this is a boolean equation (mathematical notation)

Circuit notation :



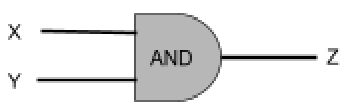
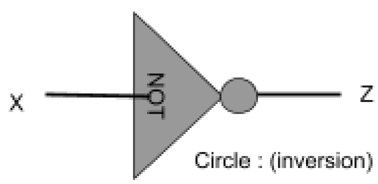
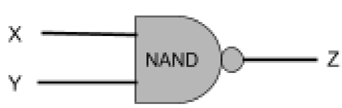
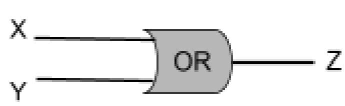
Combinational Gate Abstraction :

1) Adheres to the static discipline.

2) Outputs are a function of inputs alone. We can figure out the output for a gate simply by looking at the inputs at that instant in time.

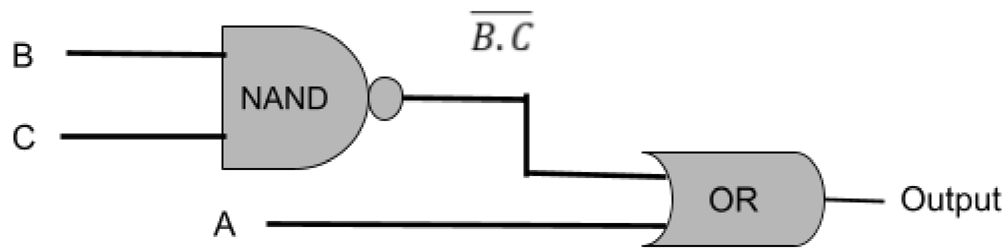
If a gate adheres to these two steps, then it falls under the combinational gate abstraction. Digital logic designers do not have to care about what is inside the gate with this abstraction.

Logic Gates.

Gate	Circuit Notation	Boolean equation	Truth table
AND		Z = X.Y	X Y Z 0 0 0 0 1 0 1 0 0 1 1 1
NOT		$Z = \overline{X}$	X Z 1 0 0 1
NAND	 (Dot for inversion) -(inverse the AND Gate values)	$Z = \overline{X.Y}$	X Y Z 0 0 1 0 1 1 1 0 1 1 1 0
OR		Z = X + Y	X Y Z 0 0 0 0 1 1 1 0 1 1 1 1

* Note that if there was no circle in the NOT Gate circuit notation, it would be just a buffer, which takes an input X and produces the output X.

Implementing $A + \overline{B.C}$:

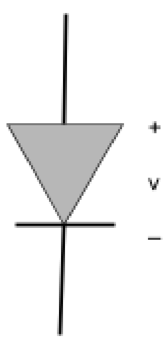


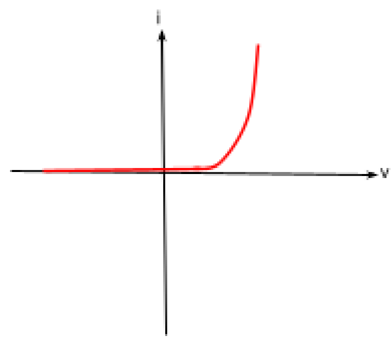
Trivia Question 3.

Is a diode linear?

$$I = I_s\left(e^{\frac{V_d}{V_t}} - 1\right)$$

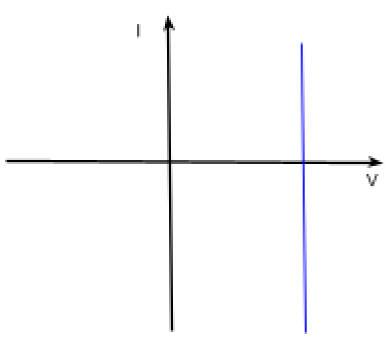
A diode has a unique v - i characteristic. When the voltage is negative there is no current allowed to flow. But after 0 volts, there would be a current and at a threshold voltage, the current increases rapidly.





The diode is not a linear system. If we take a positive voltage value at the start of the graph, there would be a small current. If we take the current reading of twice that voltage it would give a much larger current.

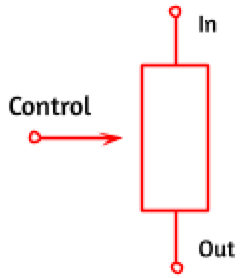
Is a voltage source linear?
The voltage of a voltage equals a constant. For example, it may be 5 volts or 1.5 volts and it is constant all the time.
v-i relationship:



Though it is a straight line, a voltage source is not a linear system. A property of any linear source has to pass through the origin.

Switch Model

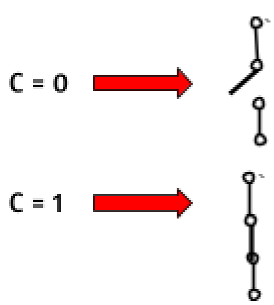
For building circuits for logic gates, we need an abstract switch device as follows ;



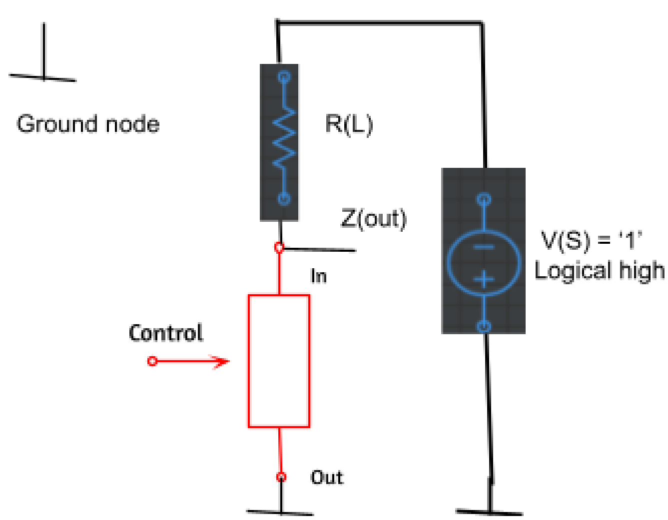
If we assume that the above is a mechanical switch, then the control pressure (mechanical pressure) on C will cause the switch to work in a given way. The switch has 3 terminals.

Under the condition where C is equal to 0, let's assume that we will get an open circuit between the input and the output (no mechanical pressure).

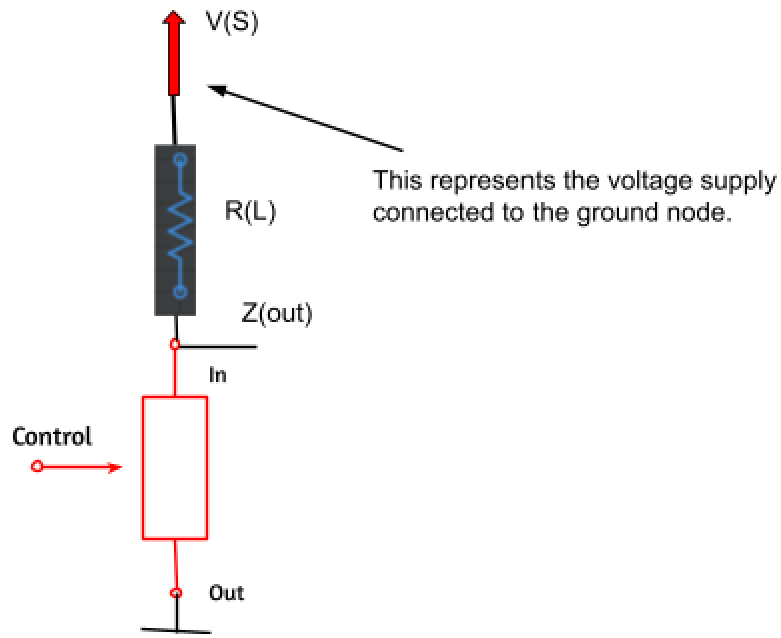
When C is equal to 1 (mechanical pressure), we get a short circuit between the input and output.



Circuit model for a NOT gate :



We can demonstrate this in a further shorthand form as shown below:



So when C is equal to 0, we know that we get an open circuit and the voltage (logical one) will appear at the output. Since no current flows down, the entire voltage V(S) will fall across this open circuit. Thus the output (Z) becomes 1 when C = 0.

When C equals 1, we get a short circuit and the output is shorted to the ground so we get 0 Volts as the output.
So when C = 0, we get a high value and when C = 1, we get a low value.

We add a resistor in a circuit with a switch and supply voltage to prevent a short circuit when the switch is closed. Without the resistor, closing the switch would directly connect the supply to the ground, causing excessive current flow and potential damage. The resistor ensures controlled current flow, keeping the node voltage at the supply level when the switch is open and at ground level when the switch is closed.

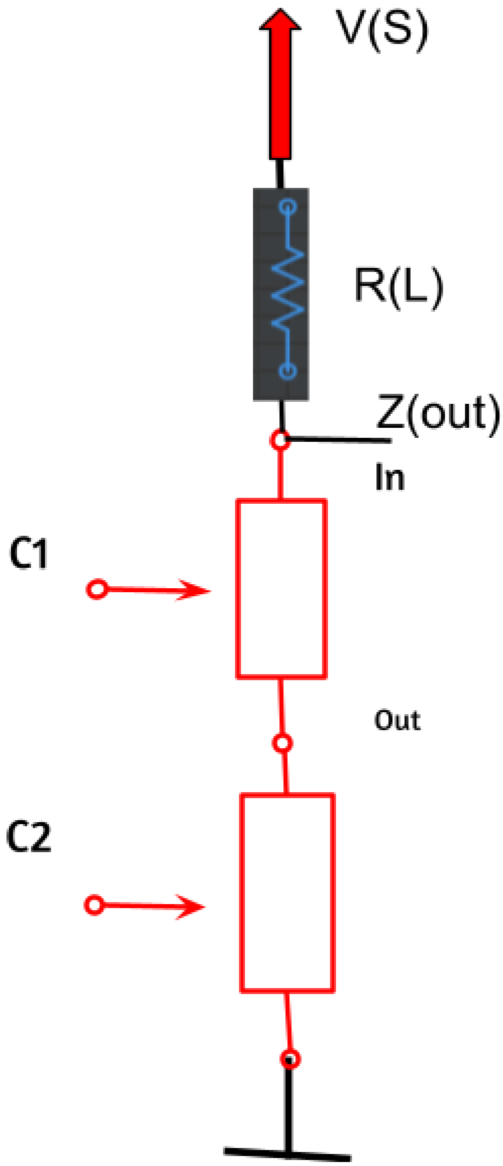
C	Z
1	0
0	1

For a NAND gate :

When C1 and C2 are equal to 0, we know that we get open circuits and the voltage will appear across the open circuit. Since it is a high value, we get a 1.

When C1 is 0 and C2 is 1 or when C1 is 1 and C2 is 0, then we get a short circuit and an open circuit in either of the switches and this gives us a logical high which is a one(1).

When both equals to 1, we get all short circuits leaving the output as a 0.

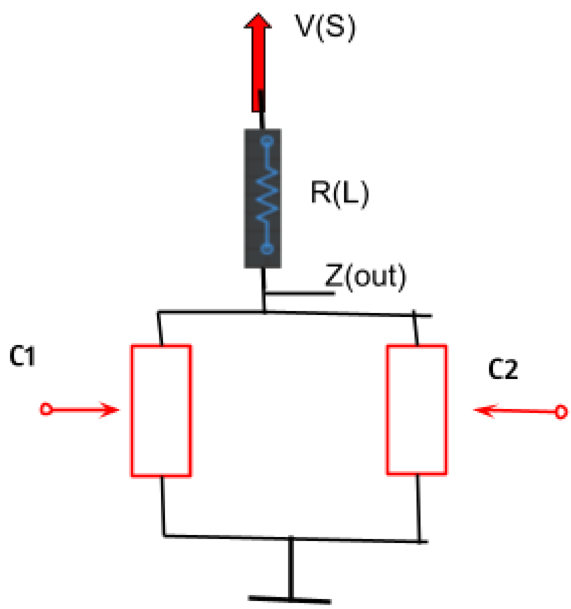


C1	C2	Z
0	0	1
0	1	1
1	0	1
1	1	0

NOR gate :

When C1 is 0 and C2 is 1 or when C1 is 1 and C2 is 0 or else when both C1 and C2 are equal to 1 we get short circuits so the resulting output would be a zero (0).

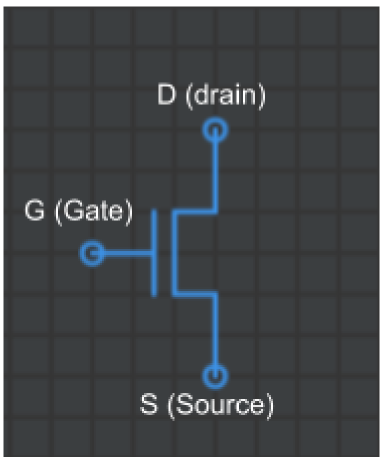
When C1 and C2 are both equal to 0 then there would be open circuits so the output would be one (1).



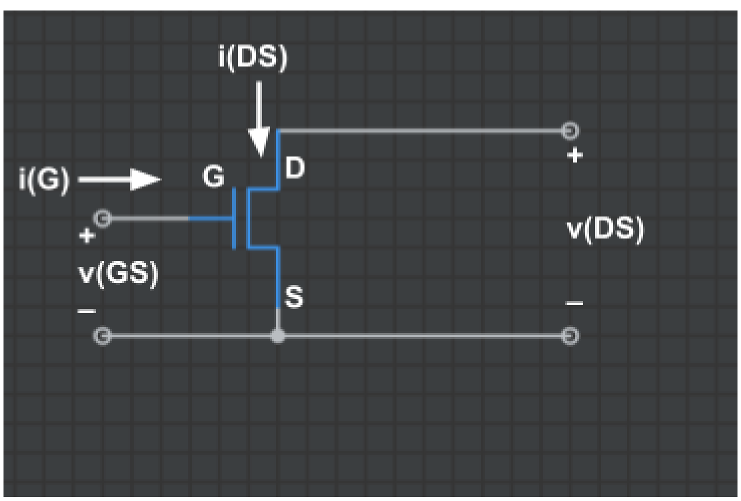
C1	C2	Z
0	0	1
0	1	0
1	0	0
1	1	0

MOSFET Device

MOSFET refers to Metal-Oxide Semiconductor Field-Effect Transistor. This is a three-terminal lumped element that behaves like a switch. It is widely used for switching and amplifying electronic signals. MOSFETs are valued for their high efficiency, fast switching speeds, and high input impedance, making them ideal for applications in digital circuits, power supplies, and amplifiers. Their small size and ability to be densely packed on integrated circuits helps to improve the performance of electronic devices



G would be the control terminal. (MOSFET is a bit more complex than switches)



The basic operation of a MOSFET (Metal-Oxide-Semiconductor Field-Effect Transistor) by modelling it as a switch with two ports:

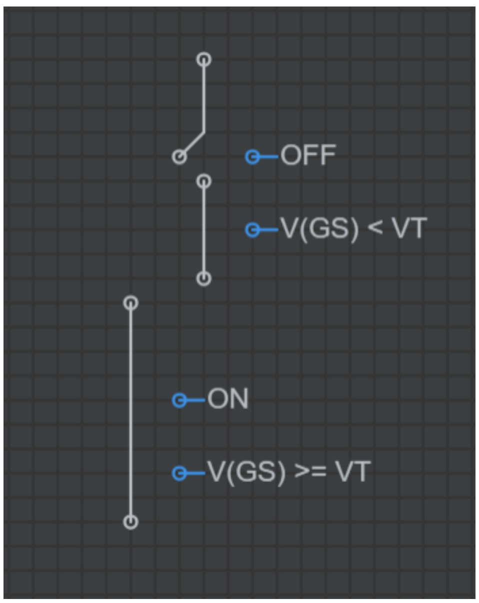
The drain-source (DS) port and the gate-source (GS) port.

When a voltage (VGS) is applied between the gate and source, and this voltage is less than a certain threshold (VT), the MOSFET behaves like an open circuit, meaning no current flows through the drain-source terminals (iDS).

In this case, VT would be for example 1 volt.

However, when VGS is greater than or equal to VT, the MOSFET acts as a short circuit, allowing current to flow between the drain and the source.

This switch-like behaviour is fundamental to understanding MOSFET operation, with the device functioning differently based on the gate-source voltage relative to the threshold voltage.



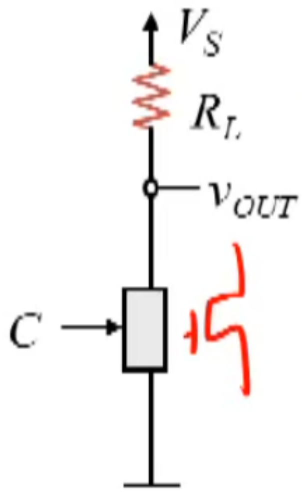
SR(Switch Resistor) Model of MOSFET

The above model that we went through shows us the Switch model of the MOSFET, which is a much simpler format. The SR model is a slightly more refined model of the MOSFET.

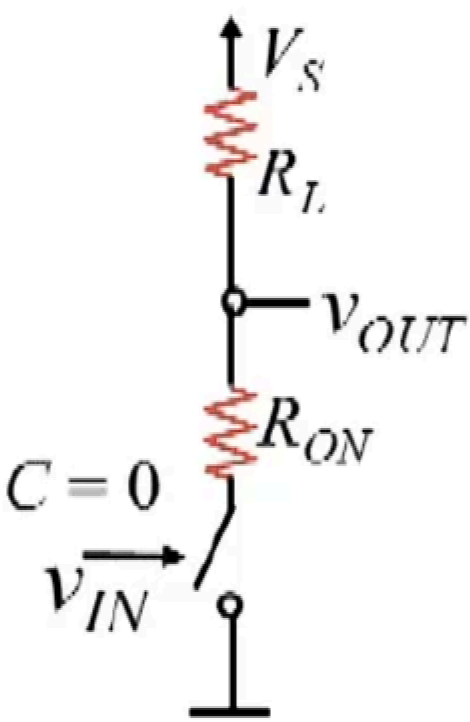
In the S model, when V(GS) was less than VT, there would be an open circuit between the drain and the voltage source, which would be the same for the SR model. (MOSFET is off)

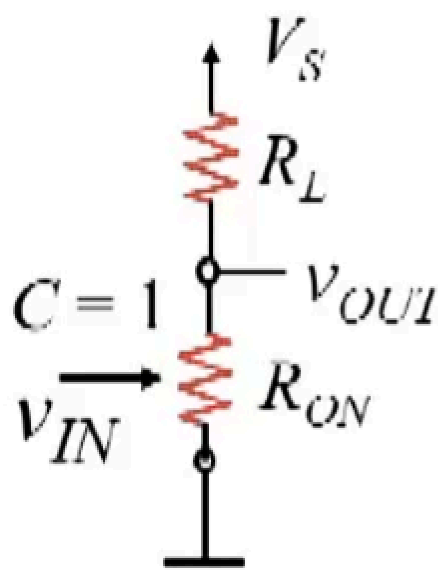
In the S model, when V(GS) is greater than VT, we get a short circuit between the drain and the source. When we apply a high-voltage at the gate to the source, there is some finite resistance between the drain and the source called R, with respect to the SR model. The MOSFET then turns on. (not an ideal switch in this case)

SR Model of MOSFET Inverter



We replace the switch, shown by the gray box, with a MOSFET.





(picture credit : MIT 6.002x basic electronics)

So, as shown above when c= 0, there is an open circuit and the MOSFET is off. When c equals 1, the MOSFET turns on along with some finite resistance, R(ON). When c is 0 the output would be 1 (logical high) and when c is equal to 1, V (OUT) is a voltage divider action between RL and R(ON).

c= 1: $$V_{OUT} = \frac{V_s \times R(ON)}{RL+R(ON)}$$

C	OUT
0	1
1	??

Basic logic behind an inverter,

To send the value 0, we need to produce V(out) < V(OL). For the receiver to interpret the corresponding output as 1, V(IN) > V(IH). Therefore one of the valid regions in the graph is V(IN) < V(IL) → V(out) > V(OH)

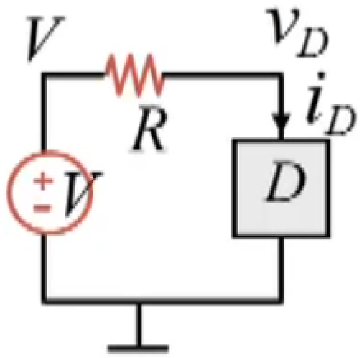
To send the value 1, we need to produce V(out) > V(OH). For the receiver to interpret the corresponding output as 0, V(IN) < V(IL). Therefore one of the valid regions in the graph is V(IN) > V(IH) → V(out) < V(OL).

Let's assume that R(ON) is much less than RL and we can make the expression,
$V_s \times R(ON)$, to be less than V(OL).

If R(ON) is much less than RL, then R(ON) can be ignored compared to RL. This means R(ON)/ RL will be significantly less than 1, and multiplying this ratio by V(S) will result in a value less than V(OL). By choosing the parameters RL, R(ON), and V(S) wisely, we can ensure that V(OUT)≤V(OL) when C=1, thus producing a correct logical 0 at the output. The key condition to satisfy is VS ×(R(ON) /(RL+R(ON)) ≤V(OL).

(R(ON) /(RL+R(ON)) ≤V(OL), which ensures a valid output when the input is in a high state.

Non-Linear Elements



$$i_D = ae^{bv_D}$$

If we were asked to perform analysis for the above non-linear circuit, we could do so using the node method. The node method is applied for linear or non-linear circuits.

$$\frac{V_D - V}{R} + iD = 0$$

$$\frac{V_D - V}{R} + ae^{bV_D} = 0$$

$$V_D = V - Rae^{bV_D}$$

Say for example, V = 1V, R= 1 Ohm, a = ¼ and b = 1 V

Substituting that to the equation gives :

$$V_D = 1 - \frac{1}{4}e^{V_D}$$

Let's substitute the following values to this equation :

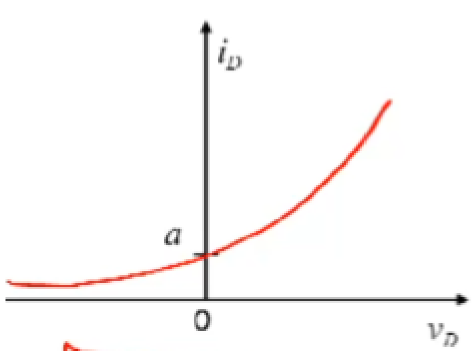
V_D (Left hand side)	V_D (Right hand side)
1 V	0.32 V
0.32V	0.65V
0.65V	0.52V
0.52V	0.58V
0.58V	0.55V
0.55V	0.56V
0.56V	0.56V

Therefore, when we can say that 0.56 V is a solution to V(D), we need to use the trial-and-error method to get the same solution for the voltage on both sides.

Next, we will look at how we can solve the above using the graphical method.

$$i(D) = ae^{bV_D}$$

The graph for this :



$$\frac{V_D - V}{R} + iD = 0$$

$$iD = -\frac{V_D - V}{R}$$

$$iD = \frac{V}{R} - \frac{V_D}{R}$$

When $V(D)$ is equal to 0:

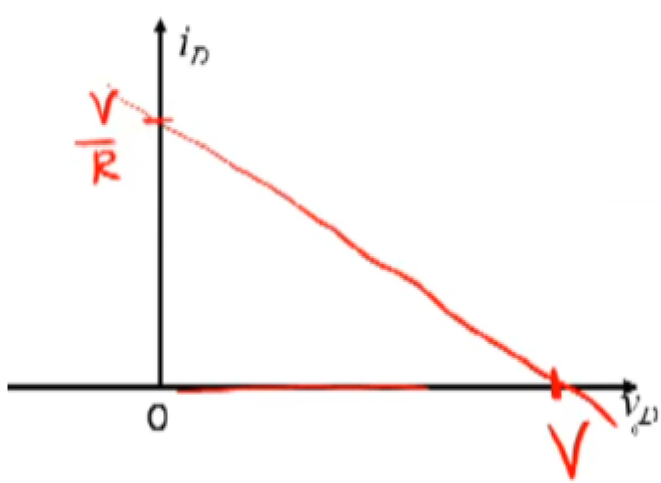
$$iD = \frac{V}{R}$$

When iD is equal to 0 :

$$0 = \frac{V}{R} - \frac{V_D}{R}$$

$$V_D = V$$

Graph :

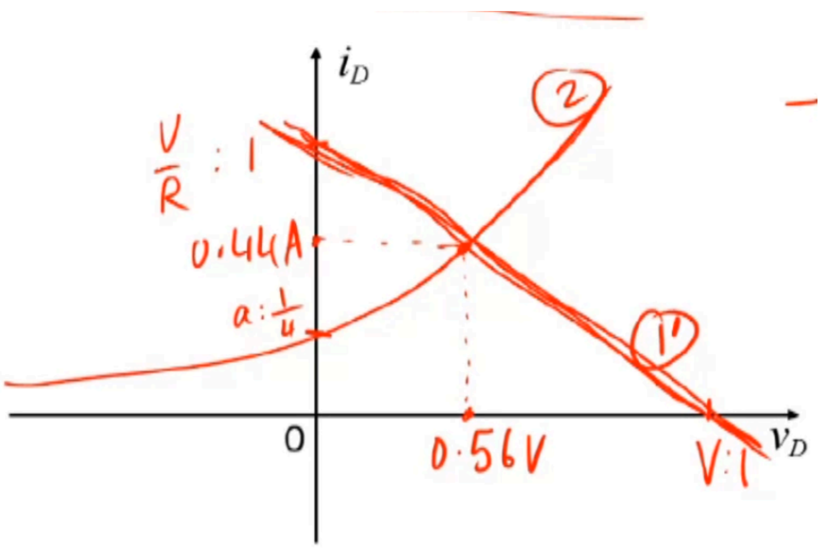


Say for example, V = 1V, R= 1 Ohm, a = ¼ and b = 1 V

Substituting that to the equation gives :

$$iD = 1 - V_D \text{------>} 1$$

$$i(D) = \frac{1}{4}e^{bV_D} \text{------>} 2$$

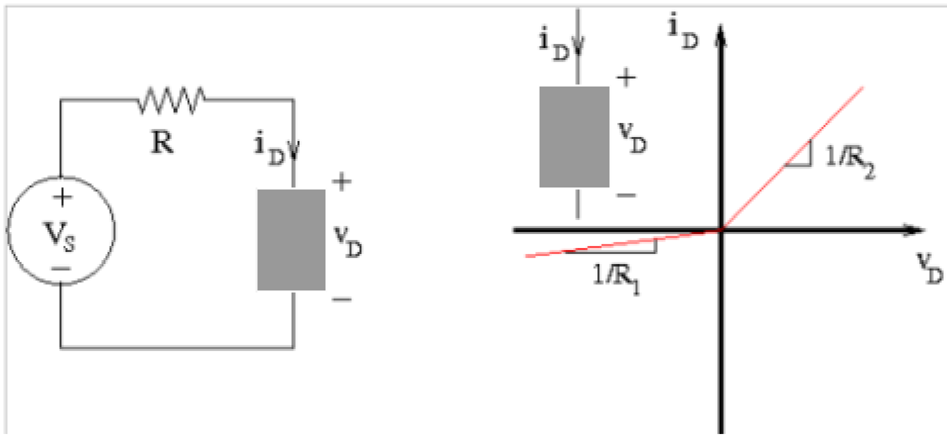


The coordinate of the intersection gives the solution. The line for the equation 1 is known as the 'load line'.

A another method would be the Piecewise Linear:
To analyze nonlinear circuits using the piecewise method, we approximate the nonlinear characteristics of components, like diodes and transistors, with linear segments. This involves breaking down the nonlinear behavior into distinct linear regions, each corresponding to different operating conditions. For each segment, we treat the circuit as linear and perform standard analysis techniques, such as using Ohm's law and Kirchhoff's laws, to solve for voltages and currents. By combining the results from each linear region, we create a comprehensive solution that approximates the overall nonlinear behavior of the circuit. This method simplifies the complex analysis by dealing with manageable linear sections.

Example :

The characteristic of the nonlinear element shown in the circuit is made up of two linear segments. The resistance of one segment is $R1 = 2.0M\Omega$ and the resistance of the other segment is $R2 = 2.0\Omega$. The resistor has a resistance of $1.2k\Omega$



If the voltage source has a strength of 8V, we can find $i(D)$ as shown below :

$since\ v > 0,\ we\ need\ to\ use\ R2,\ so\ the\ total\ reistance\ would\ be\ 2 + 1200 = 1202\Omega.$

$The\ current\ would\ be\ 8/1202\ which\ is\ equal\ to\ 0.006655574043A.$

And to find V(D) :

$V = IR$

$V(D) = 2 \times 0.006655574043 = 0.01331114809V$

If the voltage source has a strength of −8V, we can find $i(D)$ as shown below :

$since\ v < 0,$

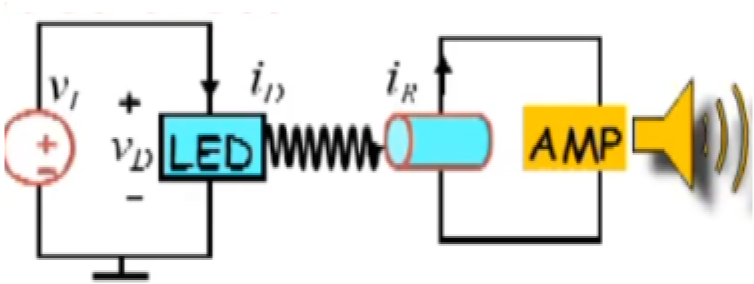
$we\ need\ to\ use\ R1,\ so\ the\ total\ reistance\ would\ be\ 2000000 + 1200 = 2001200\Omega.$

$The\ current\ would\ be\ -8/2001200\ which\ is\ equal\ to\ -0.000003997601439A.$

And to find V(D) :

$V = IR$

$V(D) = 2000000 \times - 0.000003997601439 = - 7.995202878V$



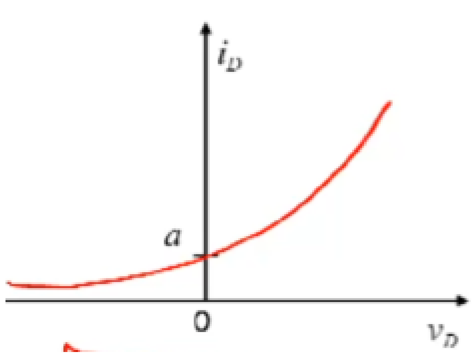
The circuit above corresponds to a gate opener circuit. $v(t)$ is converted to $i(D)$ and this is converted to light. Light falls on the photoreciver shown above in a blue cylinder and gets coveted to $i(R)$ which then gets amplified to produce sound.

The LED behaviour is much more complex and has a bit different v-i characteristic. However, to makes things simple, we will go along with the analogy.

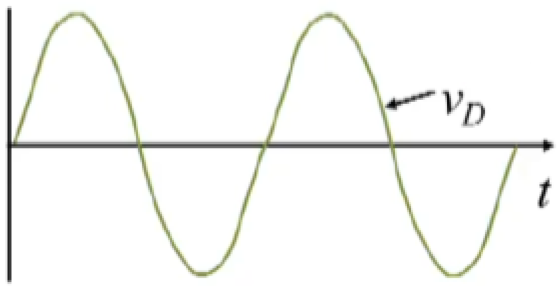
All these conversions are linear except the conversion of $v(t)$ *to* $i(D)$ as the LED device is non-linear :

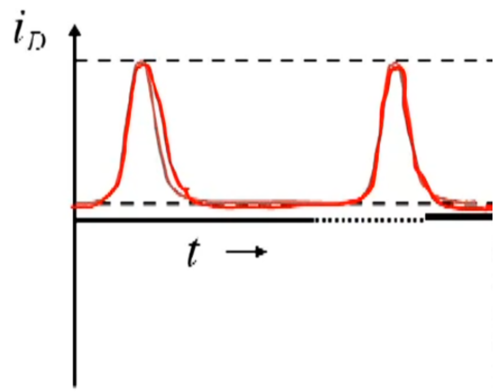
$$i(D) = ae^{bV_D}$$

Here $v(t)$ *is the same as* $V(D)$ as the 2 nodes are the same.



If we apply a sinusoidal input we can se the graphs of $v(D)$ and $i(D)$ over a time of t.

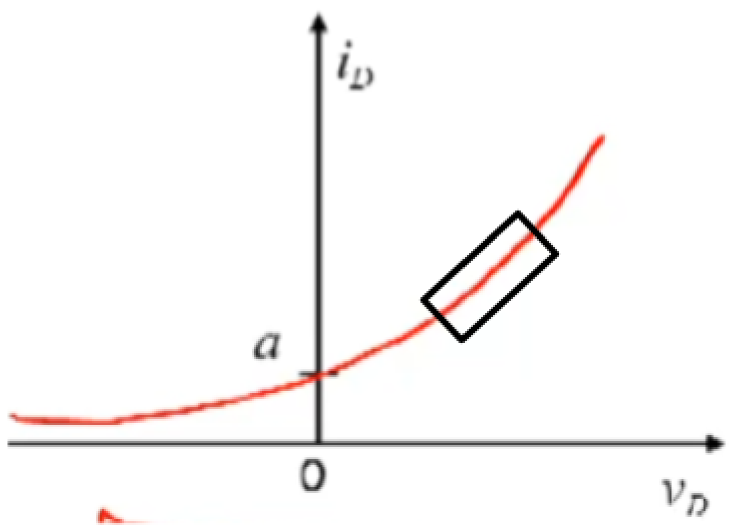




Signals are converted in a system, leading to distortion due to nonlinearity. The above graphs show how the output current is distorted compared to the input voltage.
If we pass a music signal then, the music at the other end would be distorted.

We can use the non-linear circuit to produce a linear response. This could be done using the incremental method.

We can get a linear response from a highly non-linear system by focusing on a small segment of the curve.

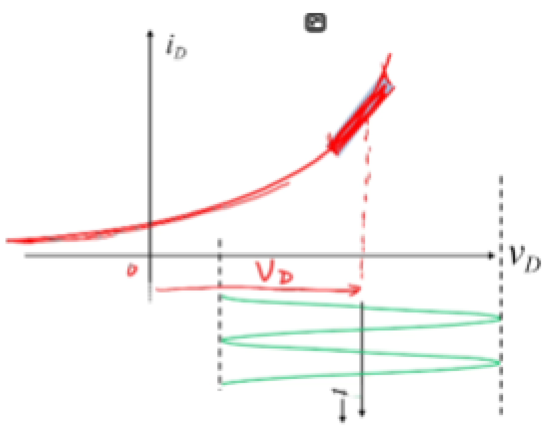


We can take a small region in the graph which seems to be linear. We need to take a small region around a specific operating point v_D and i_D.

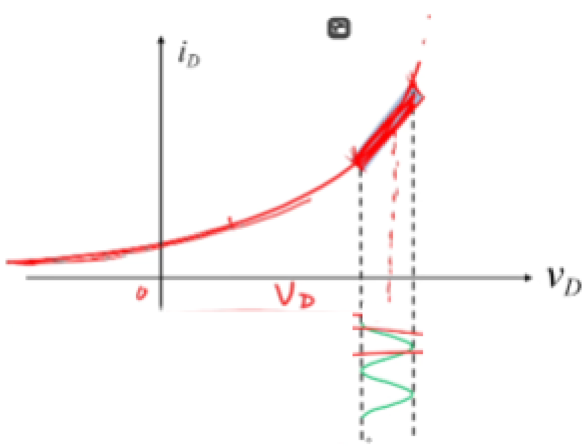
We can do this by boosting the input signal to center around v_D and then shrinking its amplitude so it only varies within this small region which seems to be linear. By this way we can assume the non-linear behavior to be linear.

Since this region of the curve is linear, the small input voltage variations, v_D produce small proportional current variations i_D allowing us to get a linear response.

Boosting the input signal :

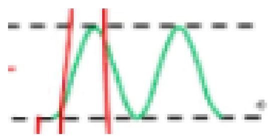


Shrinking the input voltage :



(Picture credit: MIT 6.002x)

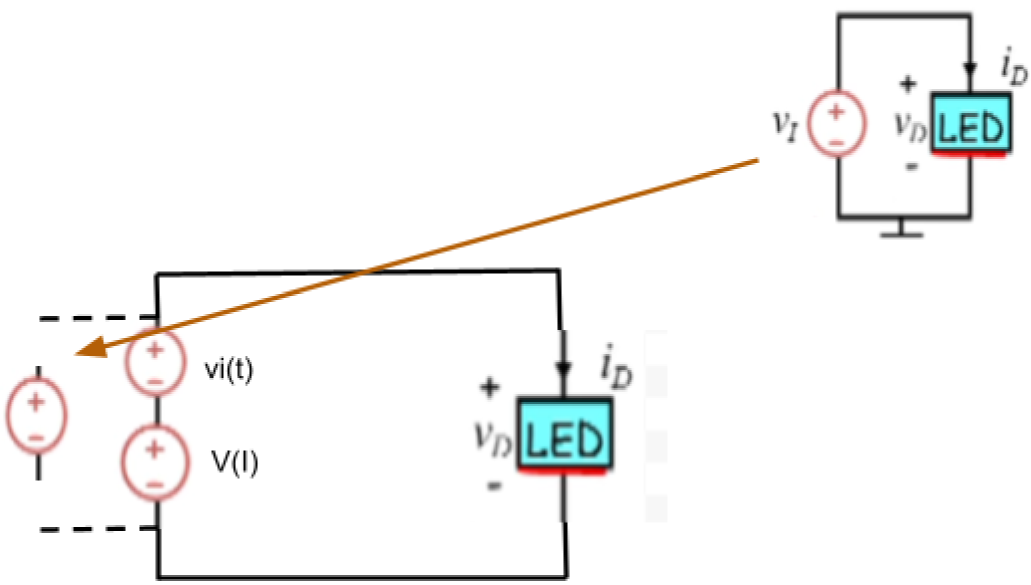
The graph for i_D *and time* :

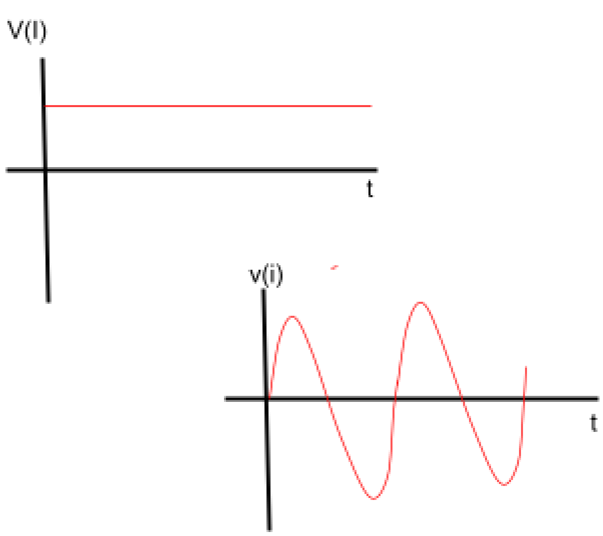


(Picture credit: MIT 6.002x)

In the above context, a DC offset involves raising the input signal by a constant value v_D, positioning it in the small, linear segment og the non-linear curve.

Let's look at how to implement it :



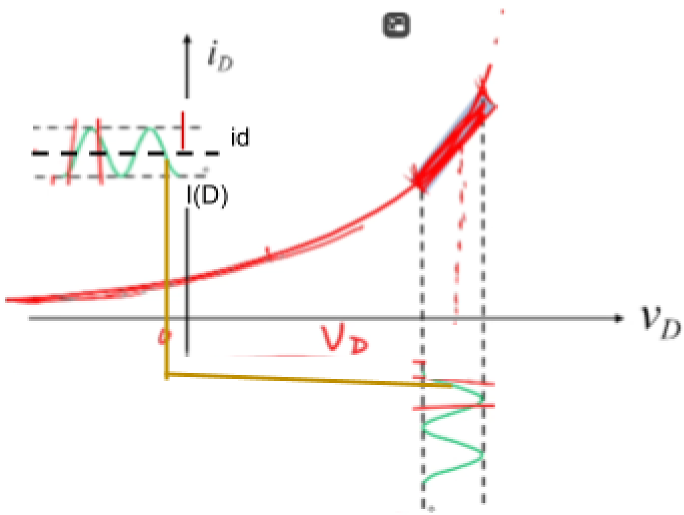


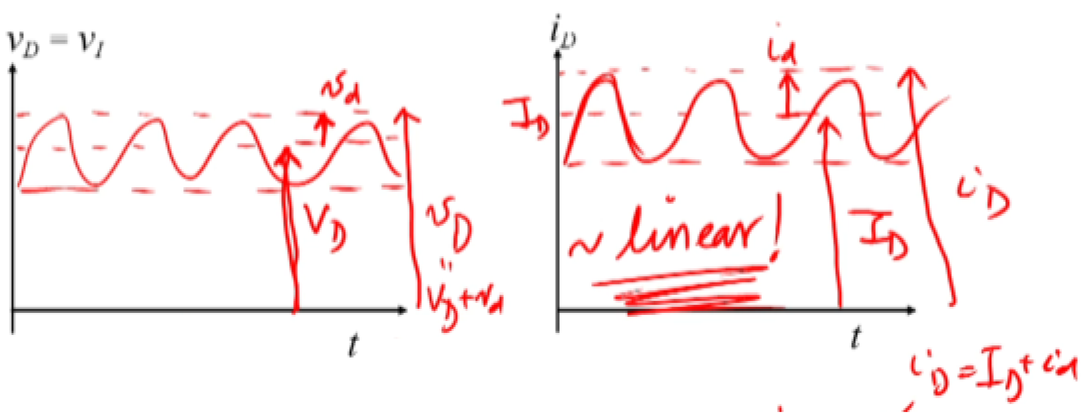
We need to add a DC source [V(I)], which gives rise to the boosting of the signal. Then we add a small AC source [vi], in series with the large DC source.

When we look at the side of the LED :

We can say that $v_D = V(I) + vi$ and the current i_D will be $i_D = I(D) + id$ at any given point in time.

$i_D = I(D) + id$ can also be represented graphically.

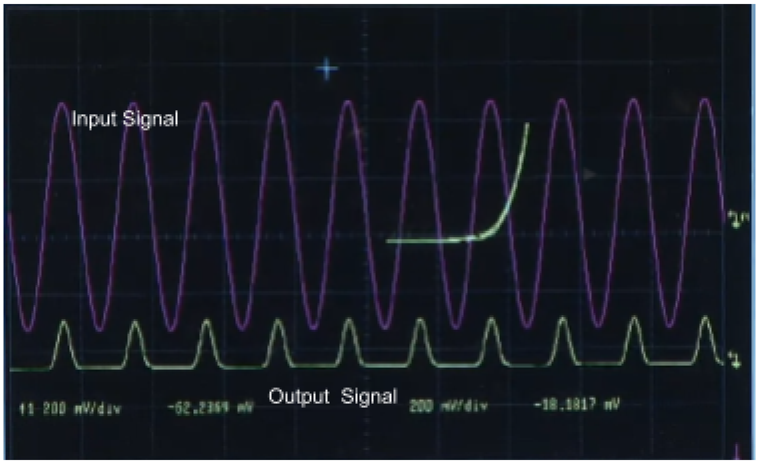




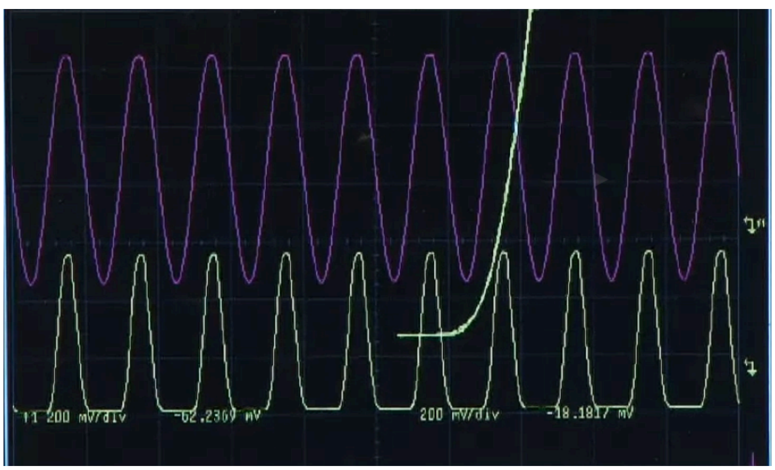
(Picture credit: MIT 6.002x)

Let's look at the oscilloscope traces for the above scenario:

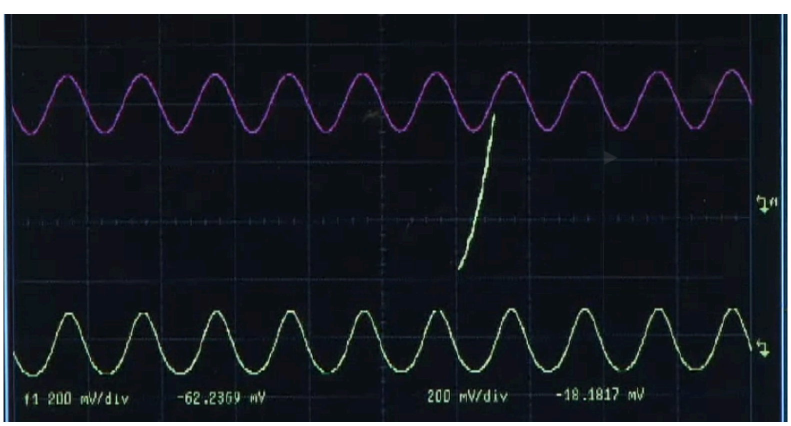
Original trace :



Boosting the original trace:



Shrinking the boosted trace :



Finally, the two traces look alike showing that the response is linear. This method can also be called the small signal method.

Let's look at the mathematical proof to show why the small signal response is linear:

Initially, we know that $i_D = f(v_D)$ and this is not linear
After getting the linear response, we know that $v_D = V_D + \Delta v_D$

Δv_D *is the mathematical representation which is the same as* v_d *or* $vi(t)$ *in the circuit. Delta sign is used to denote the small increment about* V_D *which is the large DC source.*

We can use Taylor's Expansion in order to expand $f(v_D)$ near the point of $v_D = V_D$

Taylor's Expansion Formula :

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \ldots.$$

$$i_D = f(V_D) + f'(v_D) \times (\Delta v_D) + \frac{f''(v_D) \times (\Delta v_D)^2}{2!}$$

We know that Δv_D denotes small increments about V_D so we can disregard higher order terms.

$$i_D = f(V_D) + f'(v_D) \times (\Delta v_D)$$

$$i_D \approx f(V_D) + \left.\frac{df(v_D)}{dv_D}\right|_{v_D = V_D} \cdot \Delta v_D$$

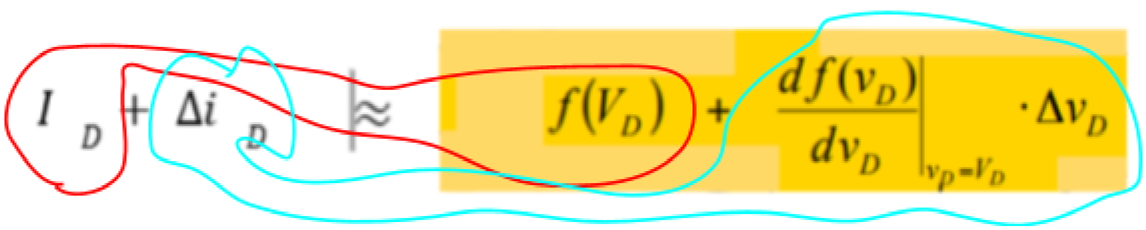
$$I_D + \Delta i_D \approx f(V_D) + \left.\frac{df(v_D)}{dv_D}\right|_{v_D = V_D} \cdot \Delta v_D$$

$$I_D + \Delta i_D = f(V_D) + f'(v_D) \times (\Delta v_D)$$

$f(V_D)$ *is a constant value as it is s fixed DC value, which doesn't change with time.*

$f'(v_D)$ *is also a constant and in fact it is the slope of the function* $f(V_D)$

$$I_D + \Delta i_D = constant + constant \times (\Delta v_D)$$



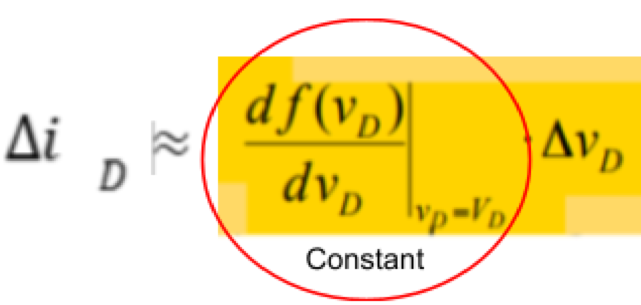
The original equation is $i_D = f(v_D)$ which is the same as the red part which is circled. The blue line shows the time-varying part of the above equation.

We can then equate the time-varying part and the DC equations.

DC part :

$I_D = f(V_D)$ —> known as the "operating point"

Time-varying part:



$$\Delta i_D \approx \left.\frac{df(v_D)}{dv_D}\right|_{v_D=V_D} \cdot \Delta v_D$$

This is a constant. This represents the slope at V(D) and I(D).

$$\Delta i_D \approx \Delta v_D$$

id vd

This proportionality shows that the incremental response is linear.

In the circuit, we used we know that $i(D) = ae^{bV_D} = f(v_D)$

$I_D + \Delta i_D = ae^{bV_D} + abe^{bV_D} \times (\Delta v_D)$

we know that Δi_D *is the same as* i_d *and* v_D *is equal to* v_d.

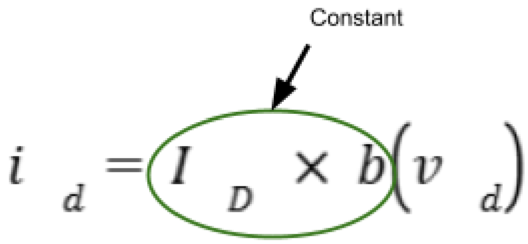
$I_D + i_d = ae^{bV_D} + ae^{bV_D} \times b(v_d)$

$I_D = ae^{bV_D}$ this is the operating point which is known to be the DC offset and also known as the bias point.

Incremental terms :

$i_d = ae^{bV_D} \times b(v_d)$

$i_d = I_D \times b(v_d)$



This shows that i_d *is approximately equal to some constant times* (v_d)
It also shows us that i_d *is proportional to* v_d
In conclusion, the small signal method is linear.

In analysing the small signal model of a circuit, we simplify complex equations by using a practical approach.

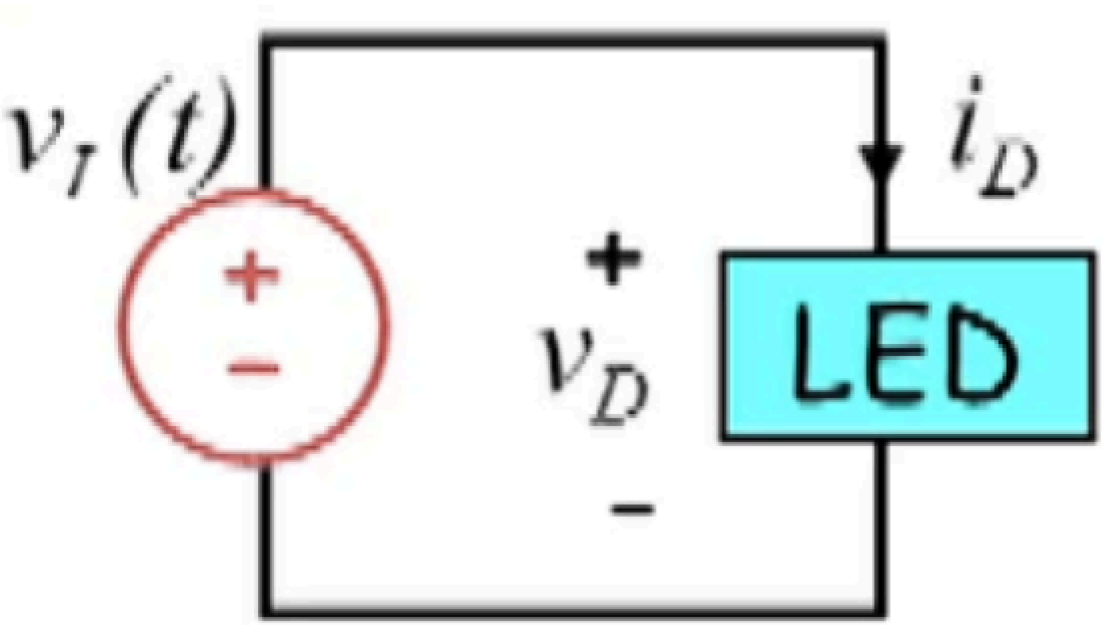
We start with a basic circuit involving a voltage source and an LED device.

We create two equivalent circuits by separating the large signal (DC bias values) from the small signal (variations).

The large signal circuit involves solving nonlinear equations, often done via analytical or graphical methods.

The small signal model focuses on the linear relationship where the current through the LED is proportional to the voltage, resembling a resistor's behaviour.

This means for small voltage changes, the LED acts like a resistor, simplifying circuit analysis for small signals.



Large signal circuit :

$$I_D = ae^{bV_D}$$

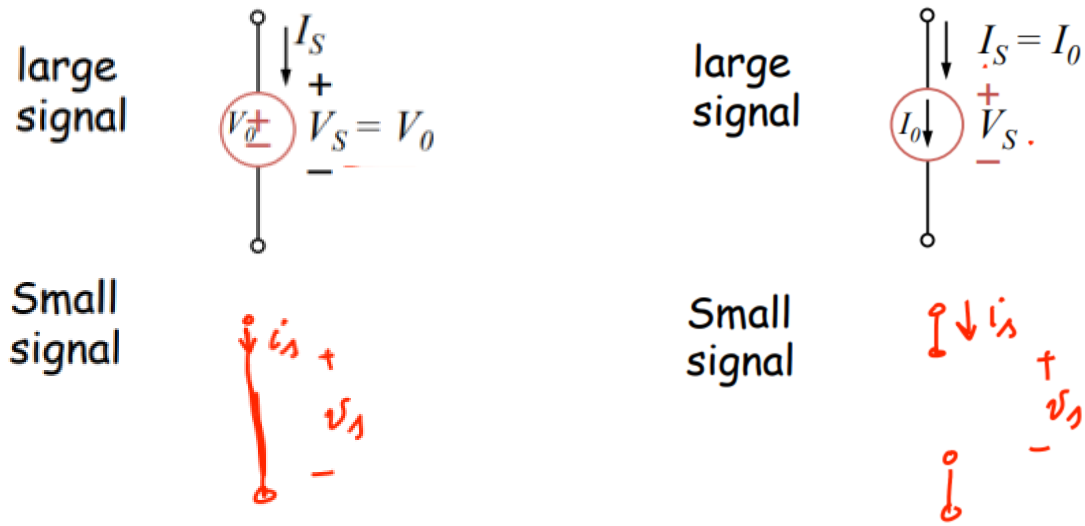
Small signal circuit :

Constant

$$i_d = I_D \times b\left(v_d\right)$$

For small signals, the device has the behaviour of a resistor.

It is important to note that a DC voltage source behaves as a short to small signals and DC current source behaves as open to small signals.

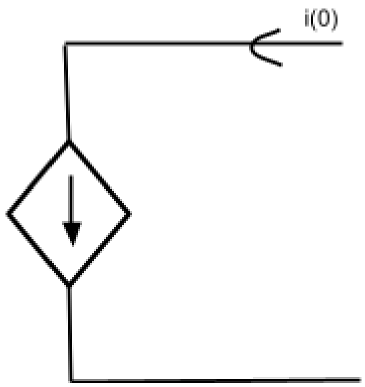


(Picture credit: MIT 6.002x)

Dependent Sources

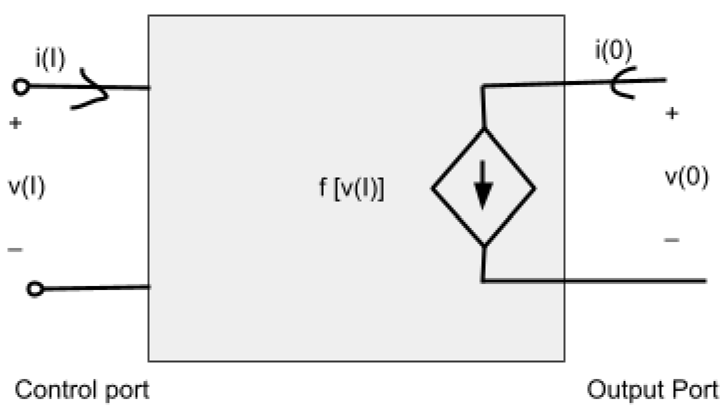
Dependent sources can either be linear or non-linear. These sources have their voltages and currents controlled by some other variables.

The symbol for a dependent current source :



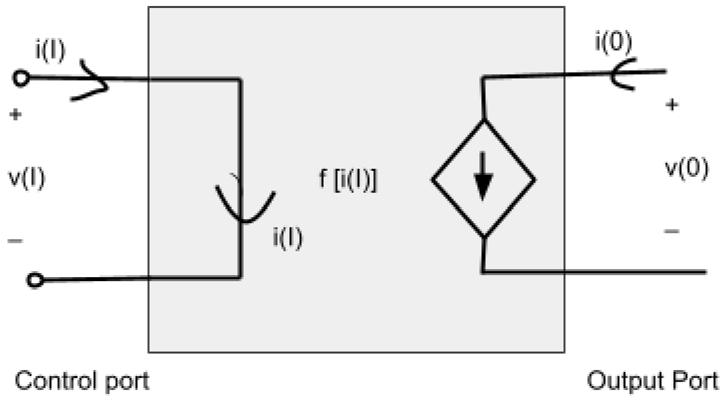
This means that the current flowing through the current source is a function of some other parameter in the circuit.

Voltage-controlled current source :



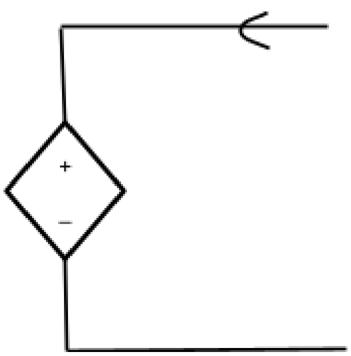
This is a 2-port device and the current is dependent on some function of the voltage v(I). The current at the output port is a function of the voltage at the input port.

Current- controlled current source :

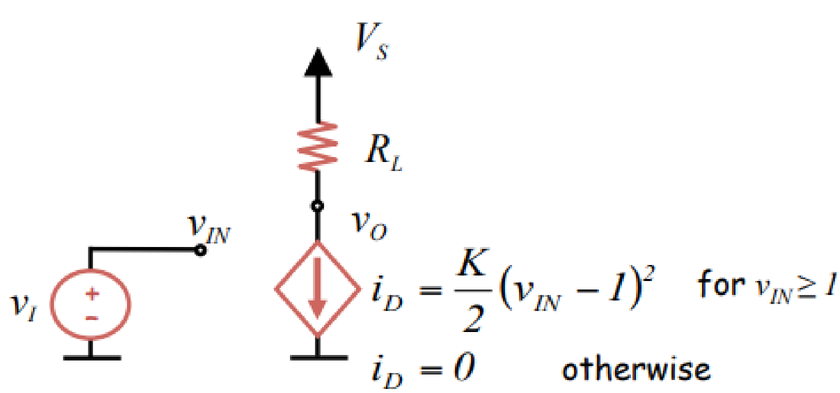


This shows that the current in the output port is a function of the current at the input port.

Dependent voltage source symbol :



Example: Solve for v_0 with respect to the ground.



$Node\ method$

$$\frac{v_0 - v_s}{R_L} + i_D = 0$$

$$v_0 = v_s - i_D R_L$$

$$v_0 = v_s - \frac{k}{2}\left(v_{in} - 1\right)^2 \times R_L \quad for\ v_{in} \geq 1V$$

$$v_0 = v_s \quad for\ v_{in} \leq 1V$$

If the values to be substituted were :

$$v_s = 10V,\ k = \frac{2mA}{V^2}, R_L = 6k\Omega$$

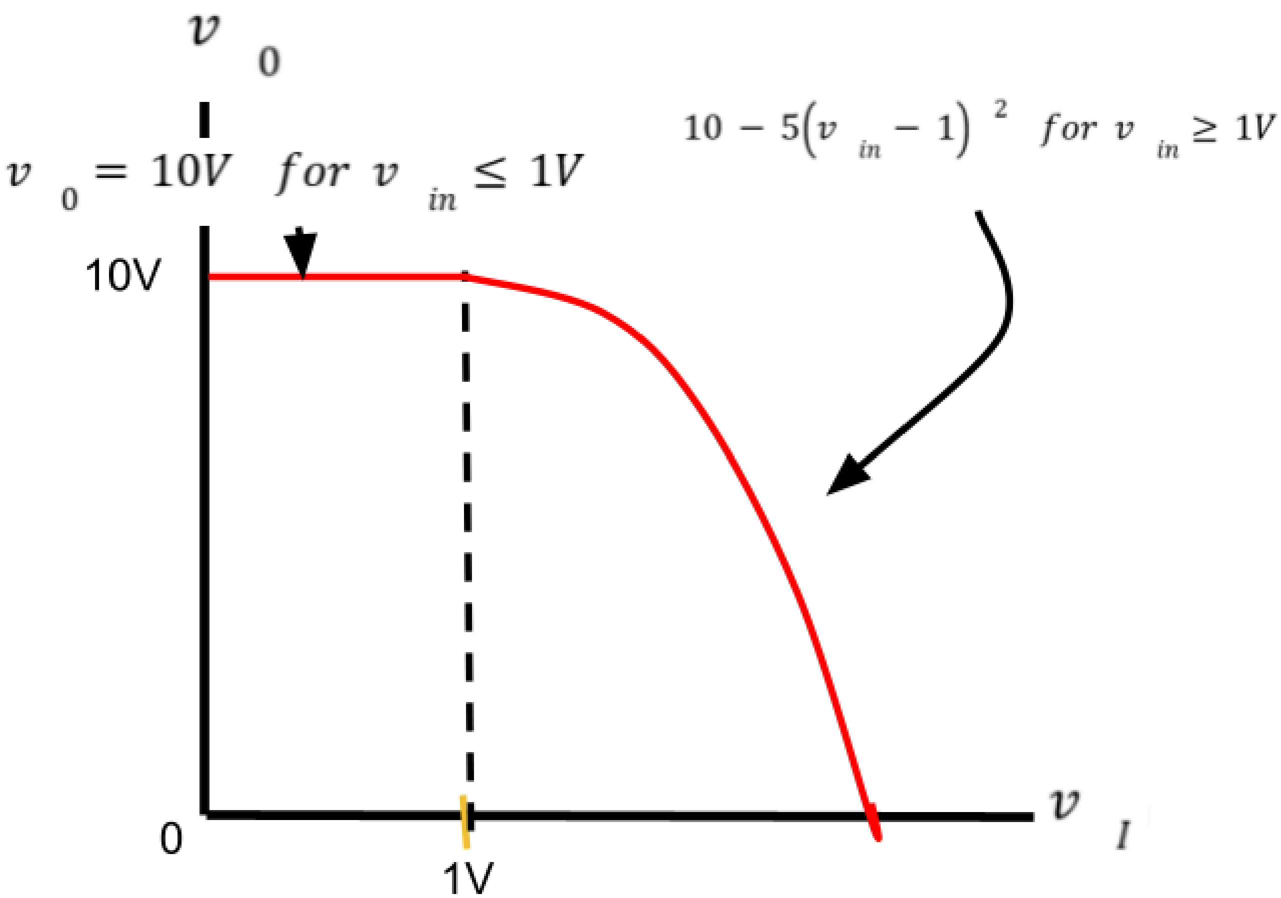
$$v_0 = v_s - \frac{k}{2}\left(v_{in} - 1\right)^2 \times R_L \quad for\ v_{in} \geq 1V$$

$$v_0 = 10 - \frac{\left(\frac{2}{1000}\right)}{2}\left(v_{in} - 1\right)^2 \times 6000 \quad for\ v_{in} \geq 1V$$

$$v_0 = 10 - 5\left(v_{in} - 1\right)^2 \quad for\ v_{in} \geq 1V$$

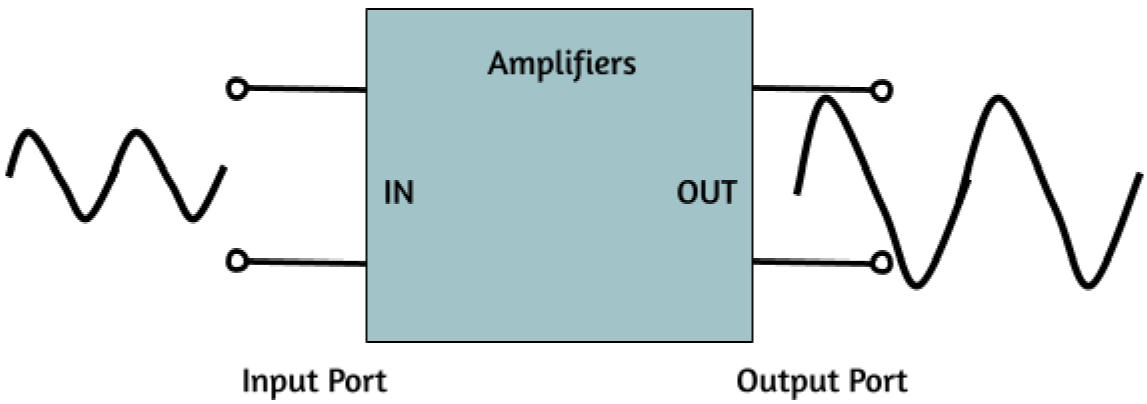
$$v_0 = 10V \quad for\ v_{in} \leq 1V$$

v_0 (vs) v_I (v_{in} is the same as v_I):



Amplifiers

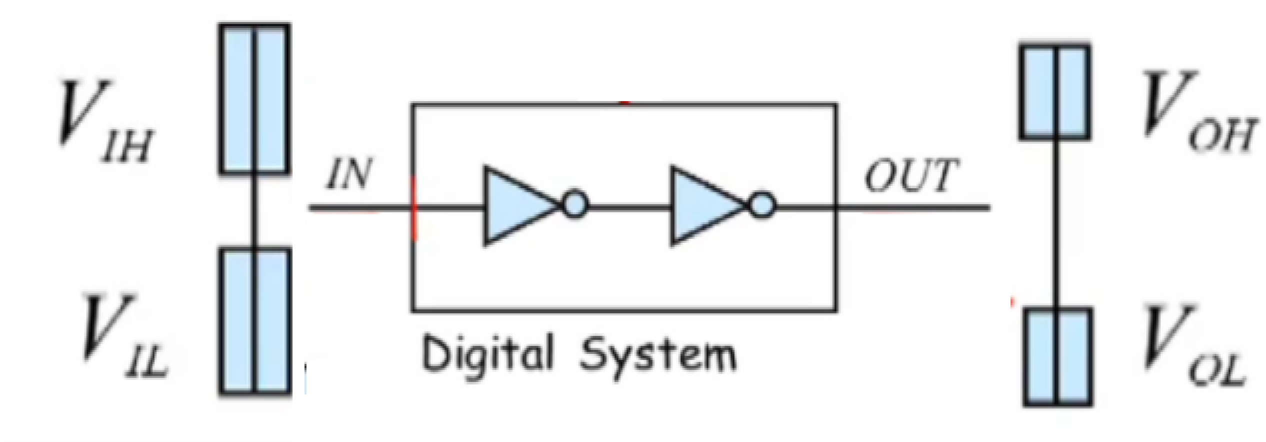
Amplifiers are electronic devices that increase the power, voltage, or current of a signal. Amplifiers work by taking an input signal and producing a stronger output signal, which retains the original signal's characteristics but with increased amplitude.



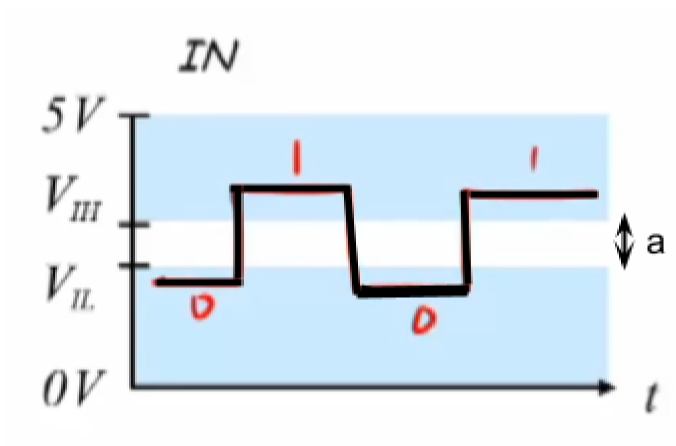
Amplification is the key to noise tolerance in communication. By amplifying a signal, it becomes easier to distinguish the signal from any added noise, making communication more effective.

In the analogue domain, amplification is needed to amplify sounds.

In the digital domain, amplification is needed to obtain thresholds for the static discipline.



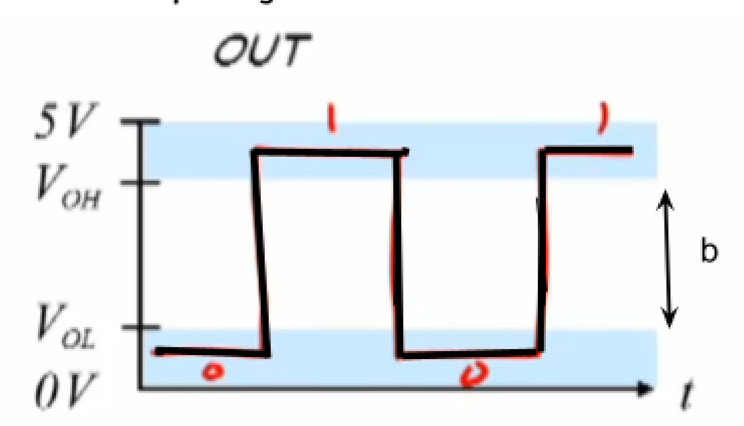
Valid Input Signal



For a logical 1 it must be greater than V(IH) and less than 5V

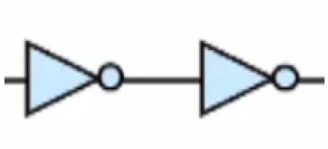
For a logical 0 it must be less than V(IL) and greater than 0.

Valid Output Signal



For a logical 1 it must be greater than V(OH) and less than 5V

For a logical 0 it must be less than V(OL) and greater than 0.



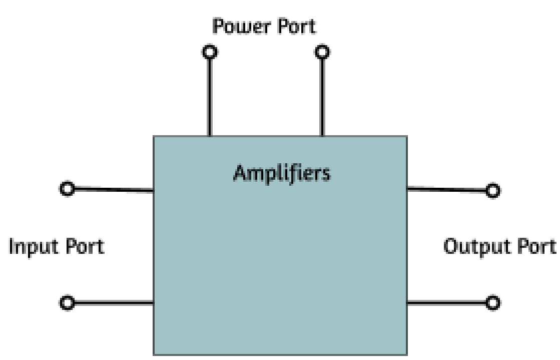
This digital system is a buffer, which produces the same signal at the output.

The output signal seems to be larger than the input signal showing that it has been amplified.

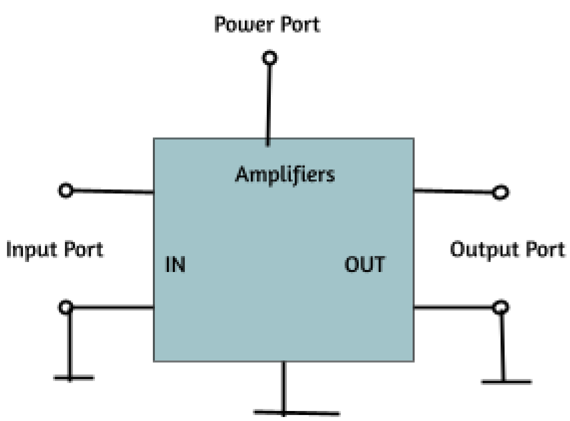
Minimum amplification applied =

$$\frac{b}{a} = \frac{V_{OH} - V_{OL}}{V_{IH} - V_{IL}}$$

An amplifier is a three ported device, which has 3 ports. Amplifiers provide power amplification.

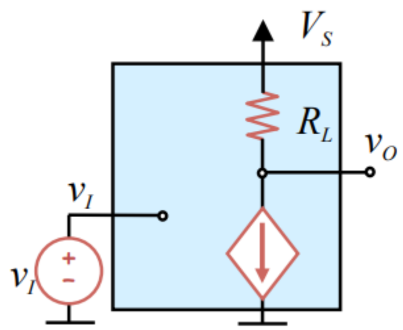


A common connection for an amplifier :



We don't show the power port often. (Terminals are connected to the ground in some amplifiers)

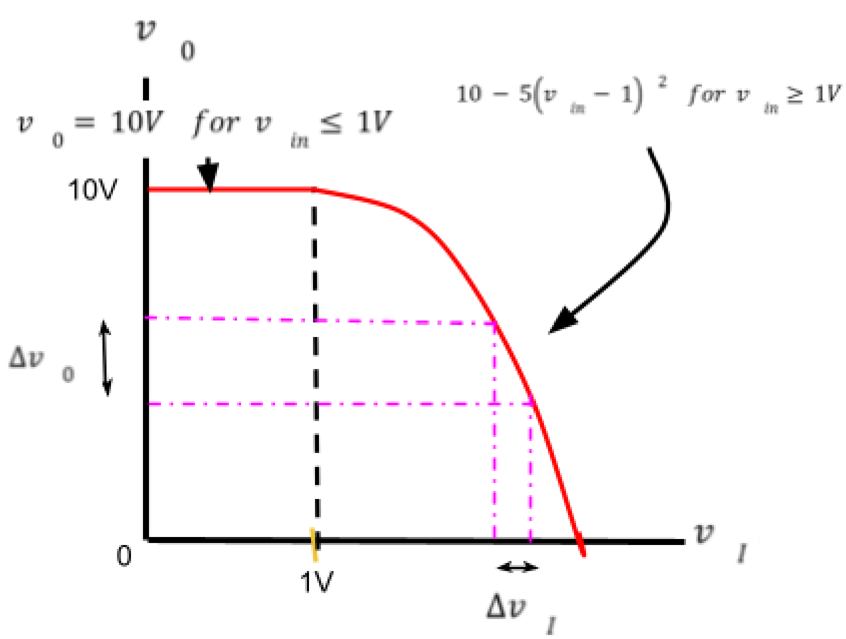
Building an amplifier :



$$i_D = \frac{K}{2}(v_{IN} - 1)^2 \quad \text{for } v_{IN} \geq 1$$

$$i_D = 0 \qquad \text{otherwise}$$

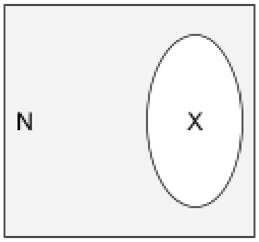
The above circuit construct could give us an amplifier.



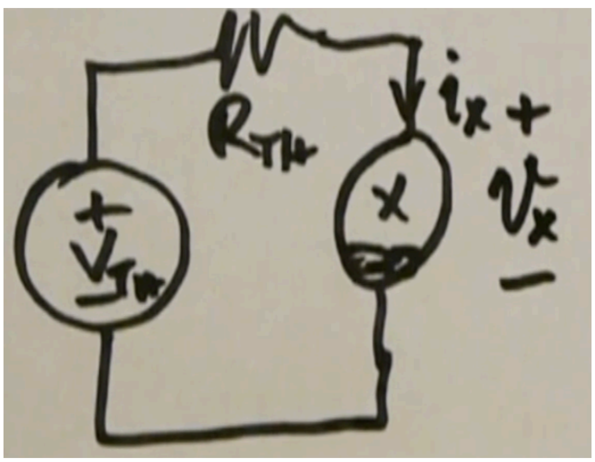
For a small change in Δv_I there is a large change in Δv_0 , showing amplification

$$\frac{\Delta v_0}{\Delta v_I} \geq 1$$

Trivia Challenge

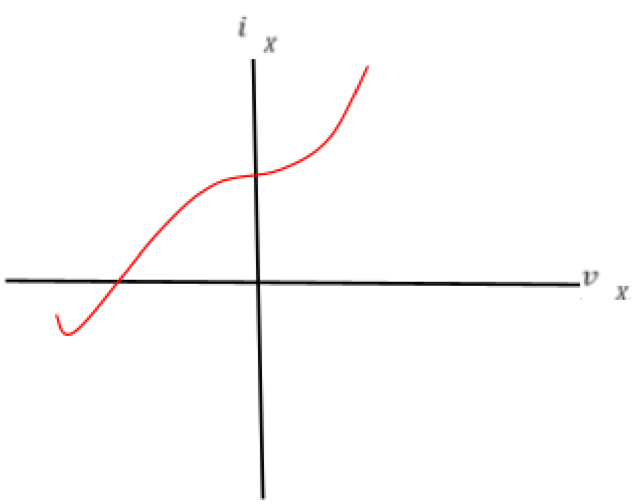


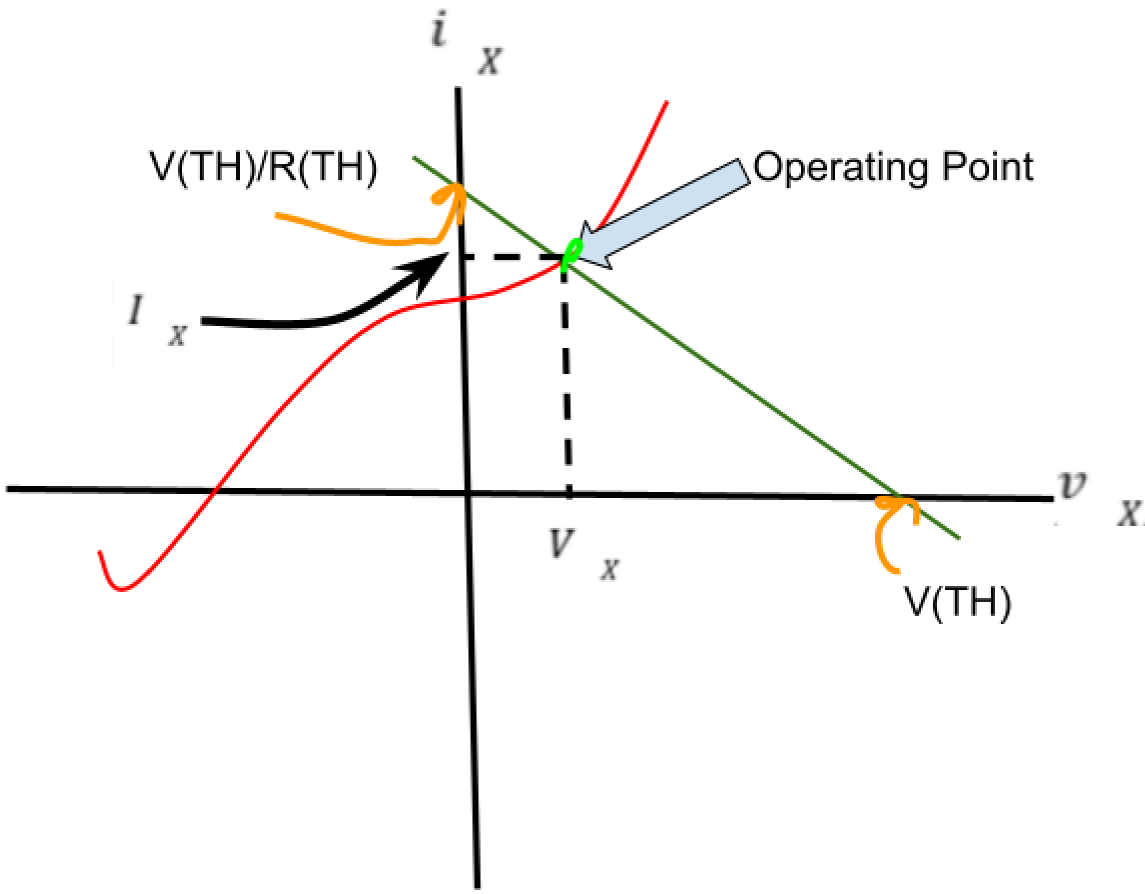
Let's say we have a two-terminal device in network N made of resistors from independent sources.
We then need to construct the Thevenin equivalent.



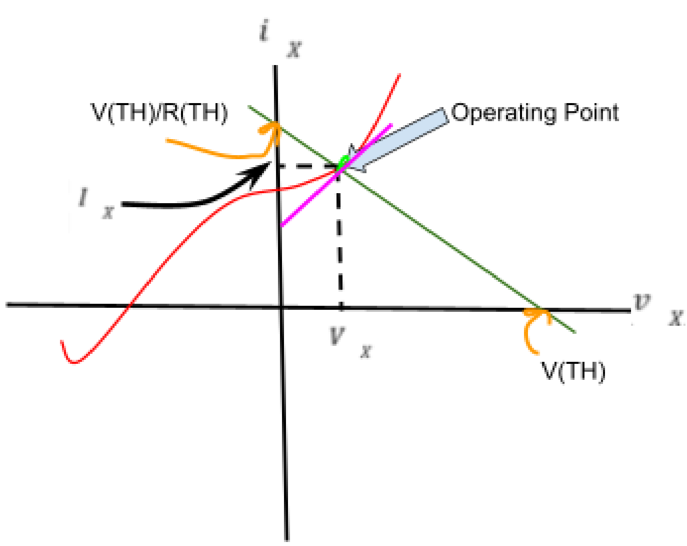
$i_X = f(v_X)$

The graph for this function would look like below:

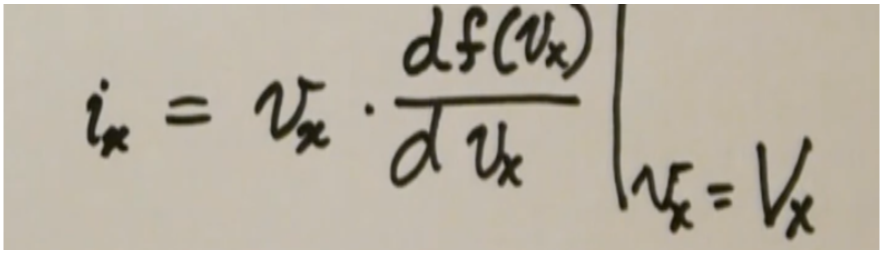




I_X and V_X are the bias current and the bias voltage respectively.



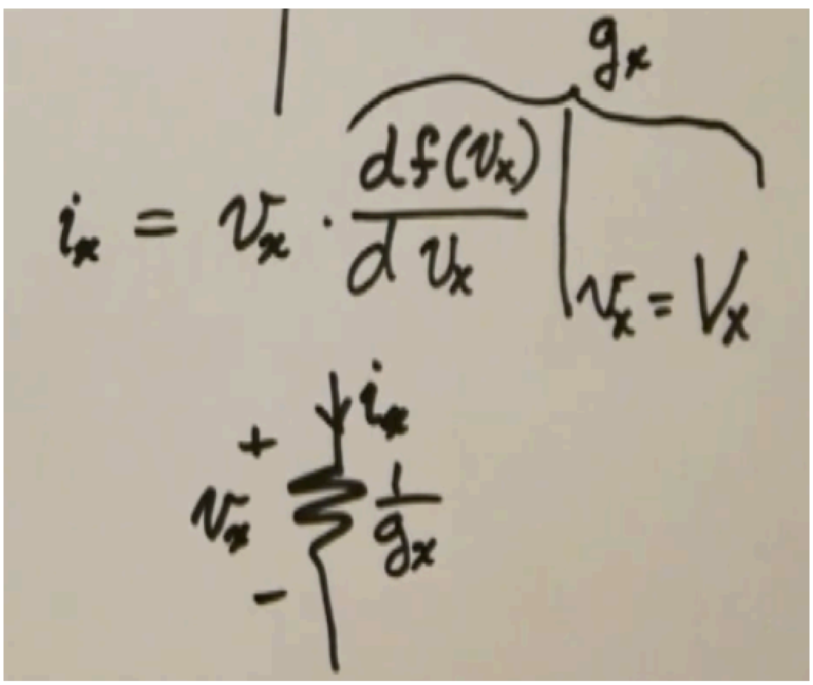
The pink straight line has something to do with the incremental world.



This incremental model is basically a resistor for this device since the derivative at the point V_X is the same as the gradient of the straight line in pink at that point. The slope of the line gives $\frac{i_X}{v_X}$.

The resistance must be the other way around ; $\frac{v_X}{i_X}$

So we do the following:



Energy

&

Thermodynamics

Contents

Basics

In Today's World Energy Consumption has increased immensely and continues daily. Being an energy analyst enables solving consumption issues and providing solutions with renewable energy sources.

Let's start off with some basic physics:

Power = $\frac{Energy}{time}$

Gravitational Potential Entergy = mgh = mass × acceleration due to gravity × height of object
Where g is approximately equal to 9.81 m/s^2

Kinetic energy = $\frac{1}{2} \times mass \times (velocity)^2$

Estimating the Total Global Primary Energy Consumption. (TGEC)

Population (POP) × per capita income/energy consumption (PCI/E) × primary energy intensity (PEI)

As we know, the TGEC increases daily and year by year. So there must be some prominent factors which could help this increase :

1) The increase in population also increases the TGEC

2) When considering the per capita energy consumption (PCI) in two different areas:

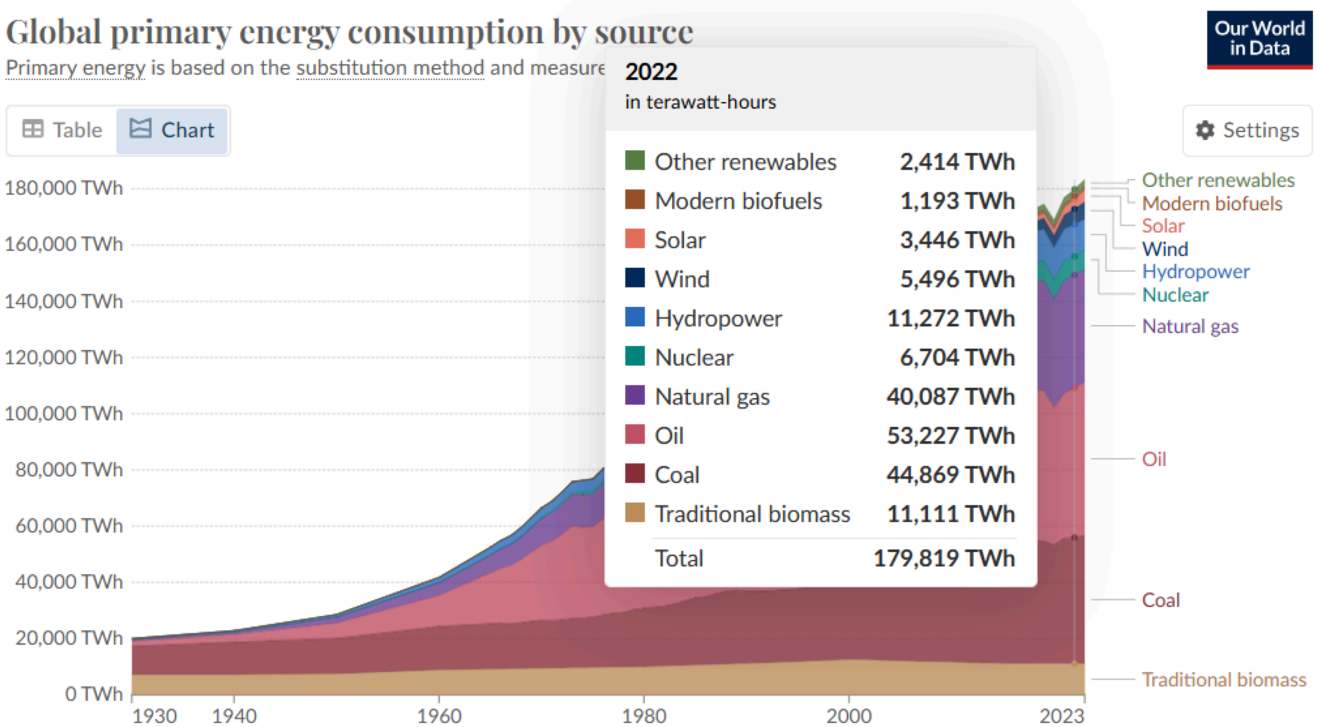
Example : Urban areas : 1200 W per captia Rural areas : 200 W per captia

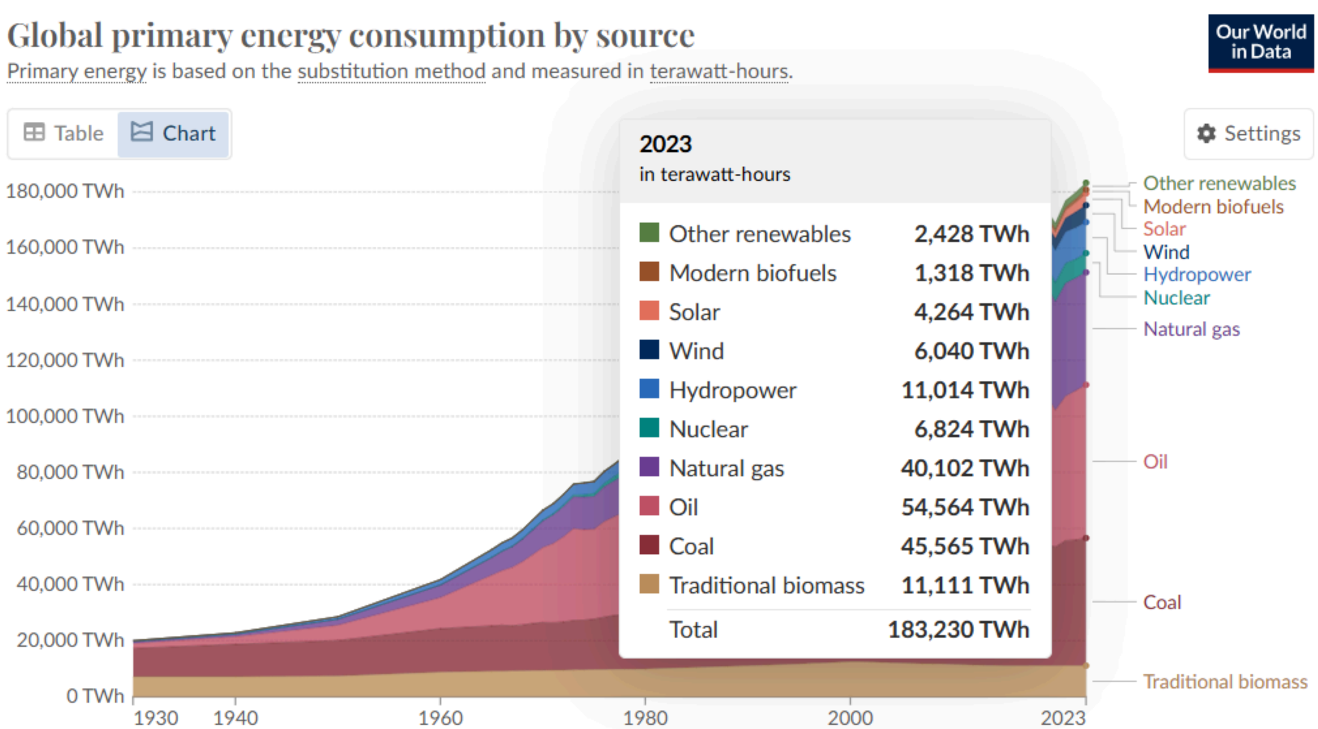
This shows that more energy is consumed in urban areas than in the rural. The increase in conversion from rural to urban drives the increase in the TGEC.

Example calculation :

The per capita income, globally averaged, was $5,500 per annum in 2014. The world population was 7 billion.
Calculate the global energy consumption (in J) in 2014 if the PEI globally averaged is 15000000 J/$GDP.

7,000,000,000 * 5,500 * 15000000 = 580,000,000,000,000,000,000 J





Source : **Data source:** Energy Institute - Statistical Review of World Energy (2024); Smil (2017)

As shown in the pictures above, the total energy consumption is greater in 2023 than in 2022. In order to analyse the percentage increase in the use of usage of renewable resources from 2022 to 2023,

Other renewables + solar + wind + Hydropower + Traditional biomass +Modern biofuels

2022 = 2,414 +3,446 + 5,496 + 11,272 + 11,111 + 1,193 =   34,932 TWh

2023 = 2,428 + 4,264 + 6,040 +11,014 + 11,111 + 1,318 = 36,175 TWh

$$34{,}932 \times \left(\frac{100 + x}{100}\right) = 36{,}175$$

Therefore x is 3.6 % to 2 significant figures. This proves to us that the world has shifted towards the usage of more renewable resources from 2022 to 2023.

Non-renewable resources :

Nuclear + Natural gas + Oil + Coal

2022 = 6,704 + 40,087 + 53,227 + 44,869 = 144,887 TWh

2023 = 6,824 + 40,102 + 54,564 + 45,565 = 147,055 TWh

$$144{,}887 \times \left(\frac{100 + x}{100}\right) = 147{,}055$$

Therefore the x is 1.5% to 2 significant figures. This shows us that the consumption of non-renewable energy has also increased.

Remarkably the percentage increase of the usage of renewable energy which is 3.6 %, is higher than the consumption of non-renewable energy.

Concept of Energy Including Forms

There are two forms of energy :

1) Organised forms of energy
2) Disorganised forms of energy

Organised forms of energy, also known as macroscopic energy, include mechanical, electrical, chemical, and electromagnetic energy.
Disorganised forms of energy, also known as microscopic energy, include thermal energy and sound energy.

It is easy to convert the organised forms of energy into the disorganised forms of energy.

We can define mechanical energy as :

$$E\ (mechanical) = Potential\ Energy + Kinetic\ Energy$$

Or as:

$$E\ (mechanical) = mgh$$

So the change in mechanical energy is calculated by:

$$\Delta E\ (mechanical) = mg\Delta h$$

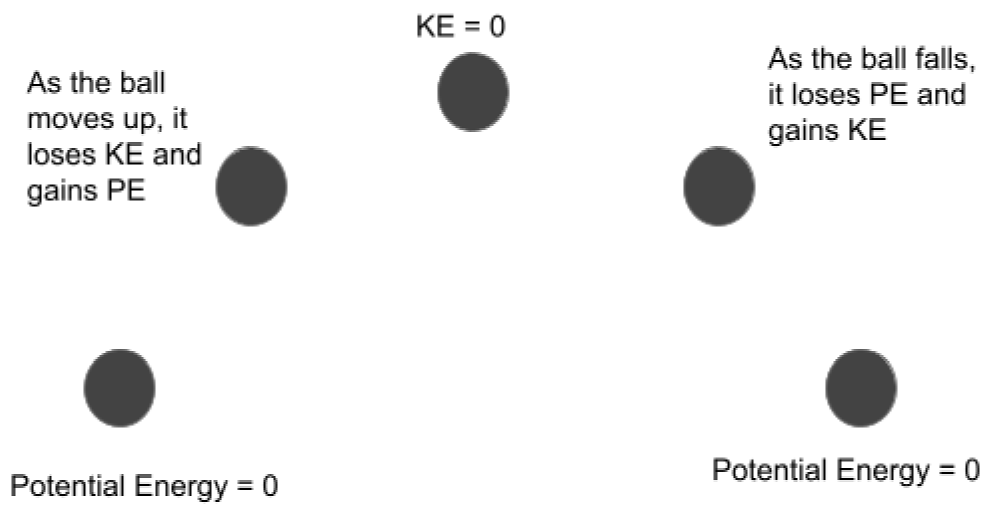
Or as:

$$E\ (mechanical) = E\ (final) - E\ (initial)$$

An important scientific advancement was the theory of 'conservation of energy', which describes that energy is neither created nor destroyed.

For example, when a tennis ball is dropped from a height, it hits the ground and rebounds to a height which is less than the height from which it was dropped.
According to the theory of 'conservation of energy', the ball should rebound to the original position.
So what would have happened?

When the ball starts falling, the gravitational potential energy converts to the kinetic energy of the ball and some thermal energy might have been dissipated due to the air resistance(force of friction acting in air against moving objects).
When it hits the ground, the kinetic energy of the ball gets dissipated to heat and sound energy.
This proves the theory of 'conservation of energy' where energy is neither created nor destroyed but transferred to other forms of energy.



The sum of the kinetic energy (KE) and the Potential Energy (PE) remains constant so if we were to find the final velocity of the ball :

$\Delta E = E\,(final) - E\,(initial)$

The change in energy is 0.

$E\,(initial) = E\,(final)$

$$\frac{1}{2}mv^{2}(final) + mgh(final) = \frac{1}{2}mv^{2}(initial) + mgh(initial)$$

$$0 + mgh(final) = \frac{1}{2}mv^{2}(initial) + 0$$

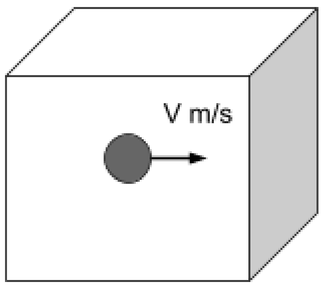
$$v = \sqrt{2gh(final)}$$

Thus by this, we can calculate the velocity of the ball.

The average energy of a molecule.

The average energy of a molecule is a function of the temperature and not of the mass of the species.

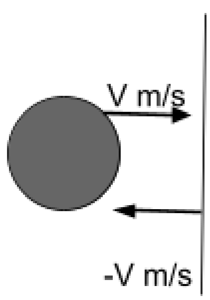
Proving this :



Sides are of L m in size

The molecule has a mass of m kg
Momentum is defined as mass into velocity

$$P\ (momentum) = mV$$



Change in velocity is V -(-V) which is equal to 2V

$$\Delta P = 2mV$$

$$F = \frac{\Delta P}{\Delta t}$$

$F = \frac{2mV}{\Delta t}$

We know that velocity is distance over time, so :

$$V = \frac{2L}{\Delta t}$$

$$\Delta t = \frac{2L}{V}$$

$$F = \frac{2mV}{\left(\frac{2L}{V}\right)} = \frac{2mV^2}{2L}$$

Pressure is force over area

$$\text{Pressure} = \frac{\left(\frac{2mV^2}{2L}\right)}{2L^2} = \frac{2}{L^3}\left(\frac{1mV^2}{2}\right)$$

If we consider the velocity in 3 perpendicular planes :

$$V_X^2 + V_Y^2 + V_Z^2 = V^2$$

$$V_X^2 = \frac{1}{3}V^2$$

Thus :

$$\frac{2N}{L^3}\left(\frac{1}{3} \times \frac{1mV^2}{2}\right)$$

Half into mass into velocity squared means kinetic energy (E)

$$P = \frac{2N}{L^3}\left(\frac{1}{3} \times E\right)$$

The volume of the container is L^3

We can then use the below equation (used of ideal gases) :

PV = nRT

Where P is the pressure Pa, V is the volume in cubic metres, n is the amount of substance in moles, T is the temperature in kelvin and finally R is the gas constant.
R is stated to be 8.31 J/mol · K.

We can say that :

nR = NK(B)

Where N is the number of molecules and K(B) is Boltzmann's constant which is :
1.38×10^{-23} J/K

$$nRT = \frac{2N}{L^3}\left(\frac{1}{3} \times E\right) \times L^3$$

$$NK(B)T = \frac{2N}{3} \times E$$

$$E = \frac{3\times K(B)\times T}{2}$$

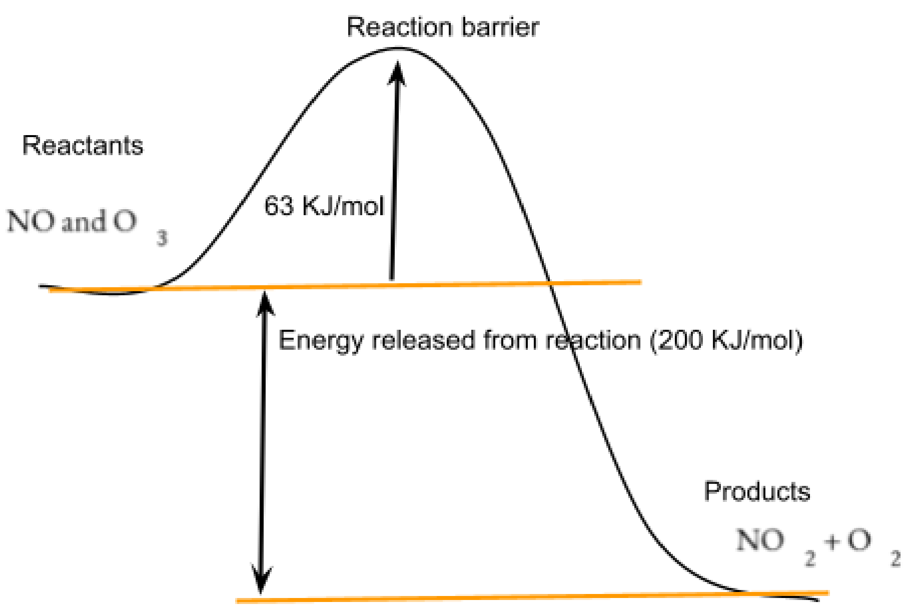
$$E = \frac{3K(B)T}{2}$$

This shows that the temperature plays an important role when determining the average energy of a molecule.

Let's take an example question to understand the concept :

The reaction coordinate diagram below shows the reactants, NO and O_3, on the left and the products $NO_2 + O_2$ on the right.

- The reaction barrier has a height of 63KJ/mol
- The energy level of the products is 200KJ/mol lower than the reactants.



For the reaction between NO and O_3, at what temperature does the activation energy equal the average kinetic energy per molecule?

$$E_A = E_{avg}$$

$$E_A = \frac{3K(B)T}{2}$$

$$T = \frac{2E_A}{3K(B)}$$

63 KJ/mol and 1 mole has 6.02×10^{23} molecules.

$$E_A = (63 \times 1000)J \times \frac{1}{6.02\times10^{23}\,molecules}$$

$E_A = 1.05 \times 10^{-19} J/molecules \ (to\ 3s.f.)$

$$T = \frac{2 \times 1.05 \times 10^{-19}}{3 \times 1.38 \times 10^{-23}}$$ K(B) is a constant with a value of 1.38×10^{-23}

$T = 5072.463768K$

$T = 5100K \ (to\ 2\ significant\ figures)$

The temperature should always be in Kelvin.

From the diagram, the activation energy will be 63KJ/mol and we need to convert the KJ/mol to J/molecules.

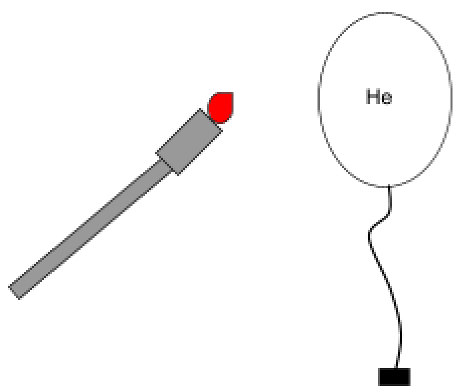
We know that 1 mole has 6.02$\times 10^{23}$ molecules which is also known as the Avogardro's constant.

Experiment with Balloons

1) Balloon filled with Helium :

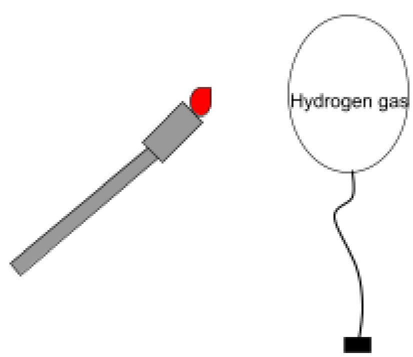
   When the flame touches the balloon with helium, the balloon pops open with a loud sound and the flame gets extinguished.

   The balloon opens by being peeled back, starting from the point where the flame touches the balloon. The balloon peels back due to surface tension.
   The candle flame is extinguished because of the helium gas escaping the balloon.



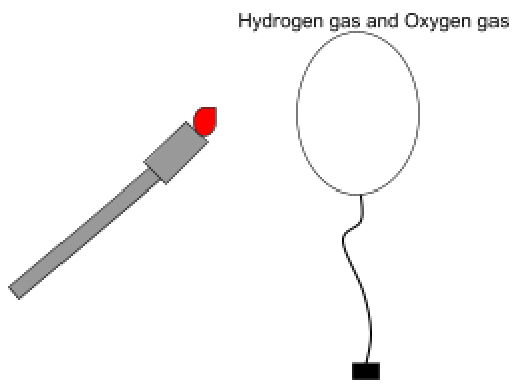
2) Balloon with Hydrogen:

   When the flame touches the balloon with hydrogen, the balloon pops open with a loud sound and the flame gets extinguished. The gas ignites as it is very flammable.
   Like in a Helium balloon, the balloon peels back due to surface tension.
   The gas ignites because Hydrogen and oxygen molecules in the air react, as they meet by diffusion, producing a fireball.



3) Balloon with Hydrogen and Oxygen ;

The balloon pops open with a very loud sound and explosive sound. A huge fireball is formed. The heat given off by the candle provides the activation energy required for the reaction that produces water from hydrogen and oxygen.
This reaction is a highly exothermic and fast reaction.



It is important to note that more energy is produced in the hydrogen-only balloon as there are more hydrogen molecules to react with he oxygen in air.

The balloon filled with both hydrogen and oxygen produces more power as it happens rapidly since the gases are already mixed.

Trivia Question 1

Assume that we have a balloon of 5 litres filled with Hydrogen. The energy released from the reaction is 285 KJ/mol. We need to find the energy.

It is important to note that a mole of gas at standard temperature and pressure is 22.4 litres per mole.

First, we need to find the number of moles :

$$\frac{5}{22.4}\ mol$$

Then, we need to multiply this value of the number of moles we found by the energy released in the reaction.

$$\frac{5}{22.4}\ mol \times 285\ KJ/mol = 64KJ\ (to\ 2\ significant\ figures)$$

We can then relate this energy release from the explosion to the energy consumption of a human each day.
Say for example on average a human consumes 2000 food calories per day.
(1 food calorie is equal to 1000 actual calories)

1 actual calorie = 4.18 Joules

$$2000 \times 1000 \times 4.18 = 8360000\ J/day$$

Finally, we need to calculate the ratio:

$$\frac{8360000}{64000} = 130\ explosions$$

Stefan-Boltzmann Law

Power output of any body = $area \times Stefan\ Boltzmann\ constant \times fourth\ power\ of\ temperature$

$P_0 = A_0 \times \sigma_{SB} \times T_0^4$

$\sigma_{SB} = 5.67 \times 10^{-8} watts\ per\ square\ meter^2 \times kelvin^4$

(W/m^2K^4)

T should be the surface temperature

Example:

We can calculate the power output of our sun which has a radius $6.96 \times 10^8 m$ and the surface temperature of the sun is 5800K

Area = $4\pi r^2 = 4\pi(6.96 \times 10^8)^2 = 6.09 \times 10^{18} m^2$

Power output of the sun = $6.09 \times 10^{18} \times 5.67 \times 10^{-8} \times (5800)^4$

$$= 3.9 \times 10^{26} W = 3.9 \times 10^{26} J/second$$

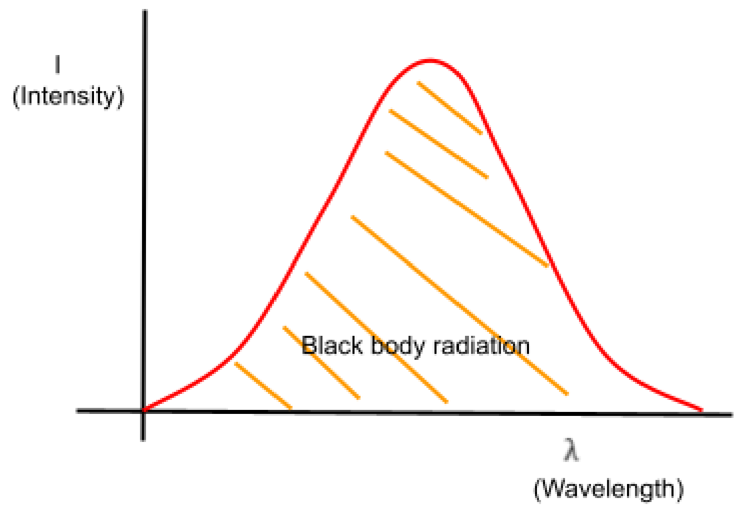
This law derives from integrating Planck's law of black-body radiation over all wavelengths, showing that the total radiative power is proportional to the fourth power of temperature.

It is essential in astrophysics for determining the luminosity of stars and in climate science for understanding the radiative heat transfer of planetary atmospheres.

In the simplicity of the Stefan-Boltzmann law:

Looking at the oscillation of electron-nuclear interaction that comprises in all materials. The electron and nuclei oscillation is intrinsic to high temperatures.

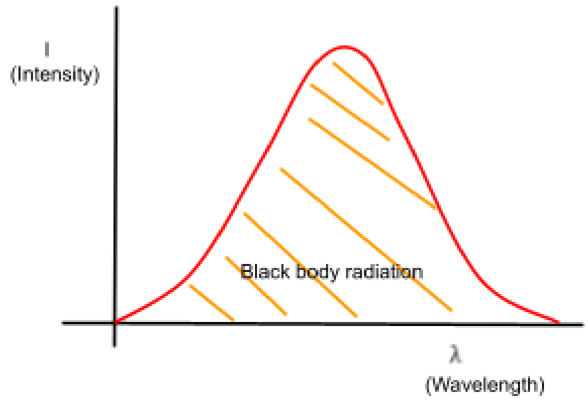
The oscillation charge leads to emission of EM radiation known as light.
Electrons oscillate around nuclei due to thermal energy. These oscillations produce electromagnetic radiation.
The intensity of this radiation increases rapidly as the temperature rises because higher temperatures cause more vigorous electron-nuclear interactions, leading to greater energy emission.



The area under the graph is the power which is calculated by ;

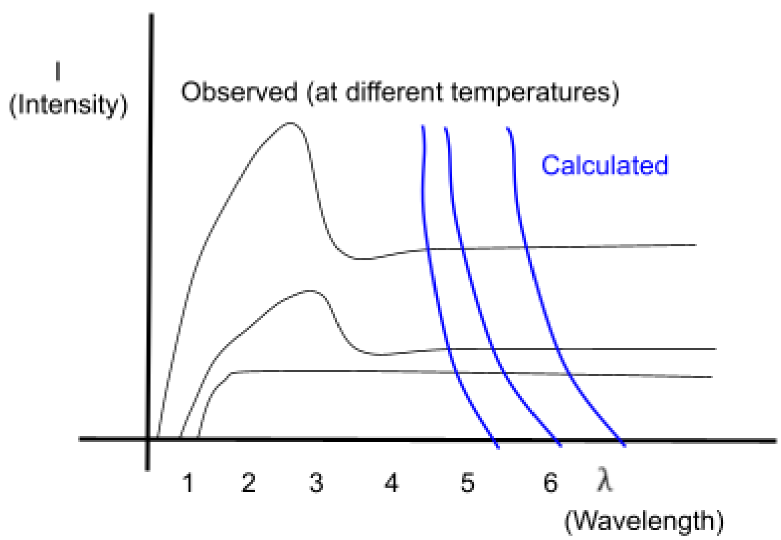
Power output of any body = $area \times Stefan\ Boltzmann\ constant \times fourth\ power\ of\ temperature$

Ultraviolent Catastrophe



During the nineteenth century, EM was considered as a wave and the behaviour of it can be calculated by a set of Maxwell's Equations.

When we apply Maxwell's Equations we get a graph like shown below :



This problem was called the 'ultraviolent Carastrophe.'
The ultraviolet catastrophe occurred when classical physics, using Maxwell's equations, predicted that a black body would emit infinite energy at ultraviolet wavelengths.
According to the Rayleigh-Jeans law, from Maxwell's equations, the energy radiated increased without limit at higher frequencies.

This contradicted experimental observations, which showed that the energy actually drops off at higher frequencies.

Max Planck resolved this issue by proposing that energy is quantized, meaning it is emitted in discrete amounts called quanta.
This new approach, known as quantum theory, accurately described black-body radiation and eliminated the ultraviolet catastrophe, showing that energy radiates in a finite and predictable manner.

Max Planck determined that the light is not a wave but it is a series of particles

** $E = hV$

$E - Energy\ of\ a\ single\ particle\ of\ light\ (Photon)$

$h - Planck's\ Constant\ (6.626 \times 10^{-34} J)$

$V - frequency$

Atomic Theory of Matter

1) All matter consists of atoms

2) They cannot be created or destroyed

3) The atoms of a given element are distinct from those of any other element, and that atoms of a given element cannot be changed by a chemical reaction.

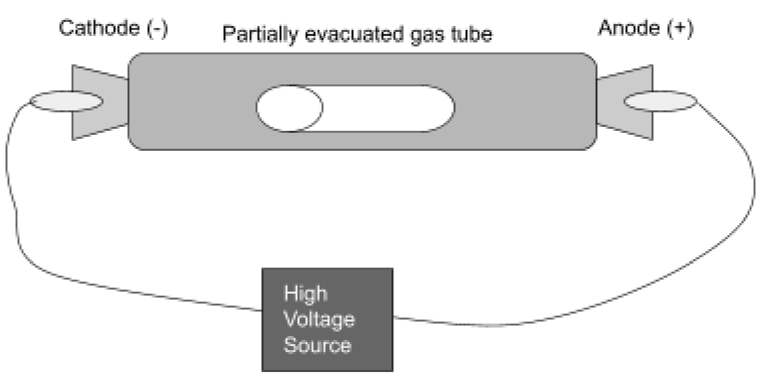
4) A chemical reaction can only change the way atoms are grouped together, that all atoms of a given element posses the same mass and same chemical properties.

5) All atoms of a given element can combine with the atoms of the other elements to form chemical compounds.

6) A given compound always has the same ratio of atoms

Experiment to show that atoms are divisible :

J.J. Thomson demonstrated that atoms are divisible by applying high voltage to gases at low pressure in a cathode ray tube.
He observed that cathode rays were deflected by electric and magnetic fields, indicating they were composed of negatively charged particles, later named electrons.
This experiment showed that atoms contain smaller subatomic particles, proving that atoms are not indivisible as previously thought.

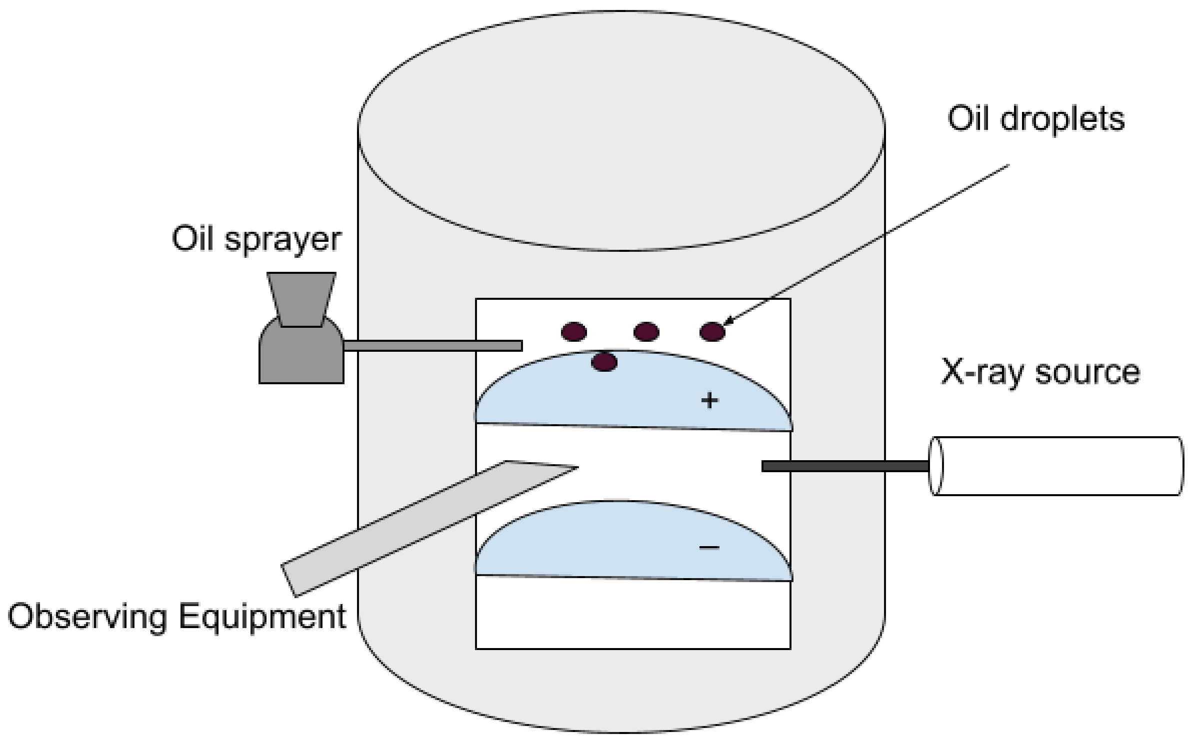


As the voltage increases the tube glows.

Determination of charge on Electron

The determination of the charge on the electron was achieved through Robert Millikan's oil-drop experiment.
By observing tiny charged oil droplets between two electrically charged plates, Millikan measured how the droplets' motion changed when exposed to an electric field.

He calculated the charge on each droplet and found that it was always a multiple of a smallest value, which he identified as the fundamental charge of the electron, approximately 1.6×10^{-19} coulombs.



1) Oil is sprayed into the apparatus in a fine mist.

2) Oil droplets fall through a hole in a positively charged plate.

3) X-rays are used to knock electrons from air which sticks to droplets.

4) Motion of the droplets is influenced by the electrically charged plates.

5) The observer controls the electric field and times the motion of droplets.

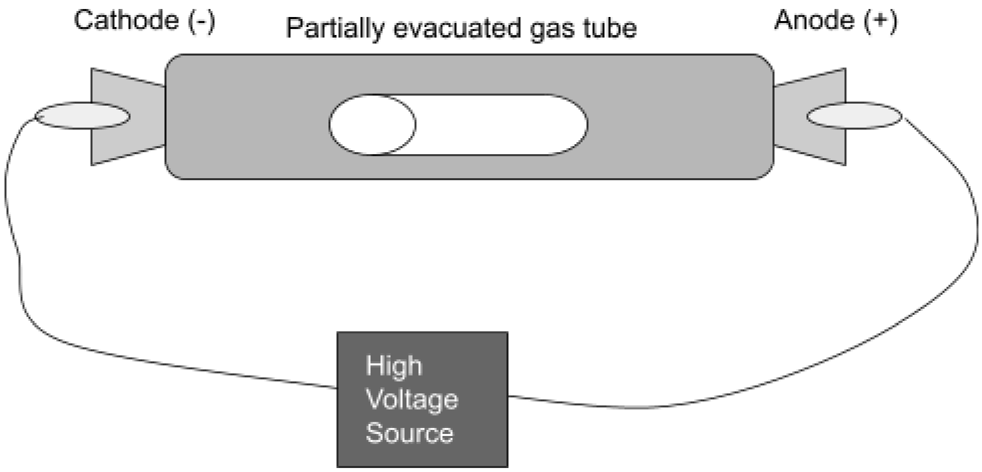
In summary :

Robert Millikan's oil-drop experiment involved spraying tiny oil droplets into a chamber and ionizing them using X-rays, causing the droplets to gain a negative charge. By observing the motion of these droplets under a microscope and adjusting an electric field between two horizontal plates, Millikan balanced the gravitational force with the electrical force, suspending the droplets in mid-air.

He measured the electric field strength needed to keep the droplets stationary, allowing him to calculate the charge on each droplet. He found that these charges were always multiples of a fundamental value, determining the charge of the electron to be approximately 1.6×10^{-19} coulombs.

This experiment provided precise evidence of the quantized nature of electric charge.

Mass of the Electron



J.J. Thomson's charge-to-mass ratio experiment involved using a cathode ray tube where he applied electric and magnetic fields perpendicular to each other and to the path of the cathode rays. By adjusting the strengths of these fields, Thomson observed the deflection of the rays and measured the extent of this deflection.
He determined that the particles (later known as electrons) had a charge-to-mass ratio of approximately $-5.686 \times 10^{-12}\ kg/C$, showing that electrons were much lighter than atoms and had a significant charge for their small mass.

We know that the charge of the electron is $-1.6 \times 10^{-19}C$.

$$-5.686 \times 10^{-12}\ kg/C \times -1.6 \times 10^{-19}C = 9.109 \times 10^{-31}\ kg$$

These experiments collectively showed that the electron is a fundamental, subatomic particle with a very small mass. The precise determination of the electron's mass was crucial for the development of atomic theory and quantum mechanics, providing key insights into the structure of matter and the nature of electric charge.

Exponential Growth and Law of 70

Exponential Growth: When a quantity Q grows at a constant percentage rate, it exhibits exponential growth.

$$\frac{\left(\frac{\Delta Q}{Q}\right)}{\Delta t} = k$$

$$\frac{\Delta Q}{\Delta t} = kQ$$

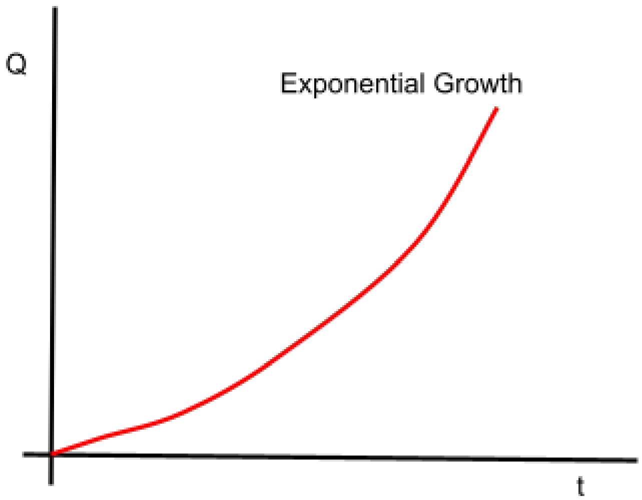
$$\lim_{\Delta t \to 0} = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

$$\int_{initial}^{final} \frac{dQ}{Q} = \int_{t=0}^{t} k dt$$

$$ln\left(\frac{Q_t}{Q_o}\right) = k \int_{t=0}^{t} dt = kt$$

$$ln\left(\frac{Q_t}{Q_o}\right) = kt$$

$$Q_t = Q_o e^{kt}$$



The final value of Q is twice the initial value

$Q_t = 2Q_o$ calculate doubling time

$$kt_2 = ln\left(\frac{Q_t}{Q_o}\right) = ln\left(\frac{2Q_o}{Q_o}\right) = ln2$$

$$t_2 = \frac{ln2}{k}$$

$$t_2 = \frac{0.693}{k}$$

Percentage growth rate :

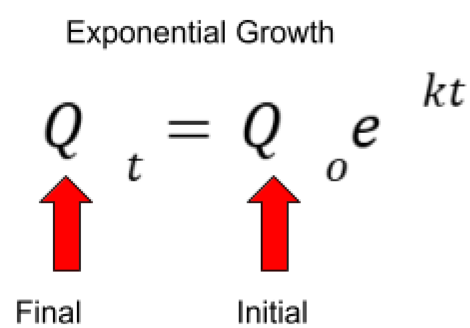
$$p = 100k$$

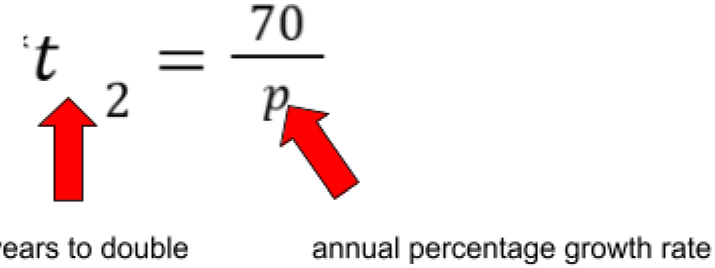
$$k = \frac{p}{100}$$

$$t_2 = 0.693 \times \frac{100}{p}$$

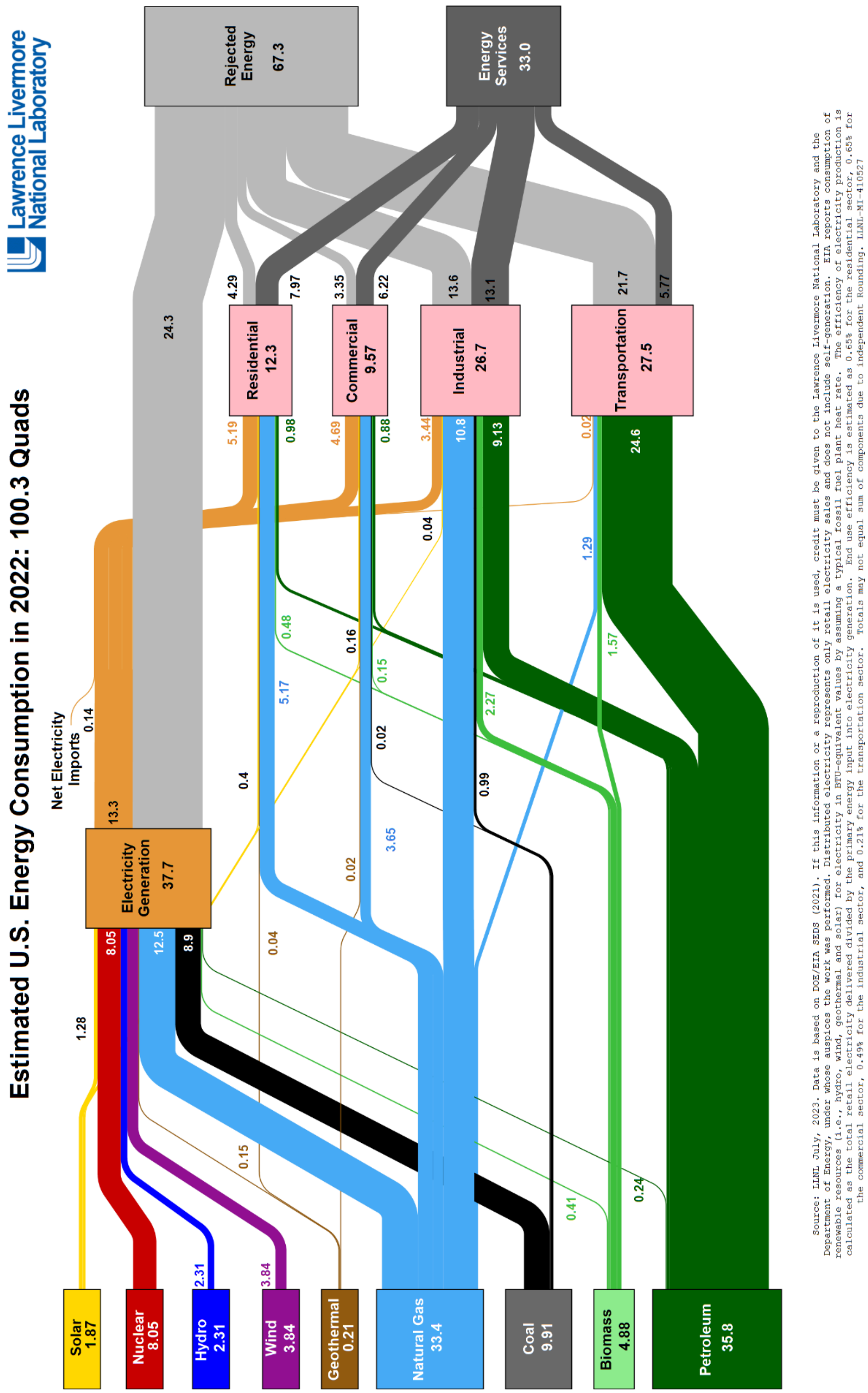
$$t_2 = \frac{69.3}{p}$$

$$**t_2 = \frac{70}{p}$$





Introduction to Thermodynamics



In the diagram above, the “Rejected Energy” is known to be the wasted energy.
The estimated U.S. Energy consumption in 2022 is 100.3 Quads.
Quand is a unit of energy which is used to determine the flow of energy through economic structures.

1 Quad is equivalent to 10^{15} British Thermal Units (BTU)

1 BTU = 1.055×10^{3} J = 1.055 KJ

Therefore :
1 quad = 1.055×10^{18} J

Percentage of overall wasted energy:

$$\frac{67.3}{100.3} \times 100 = 67\%$$

This shows us that 67 % of the energy is wasted. Thus we can use the First Law of Thermodynamics to demonstrate why 65% to 70% of the global energy generation is wasted. Thermodynamics describes the collision between energy generation and irreversible changes to the climate structure.

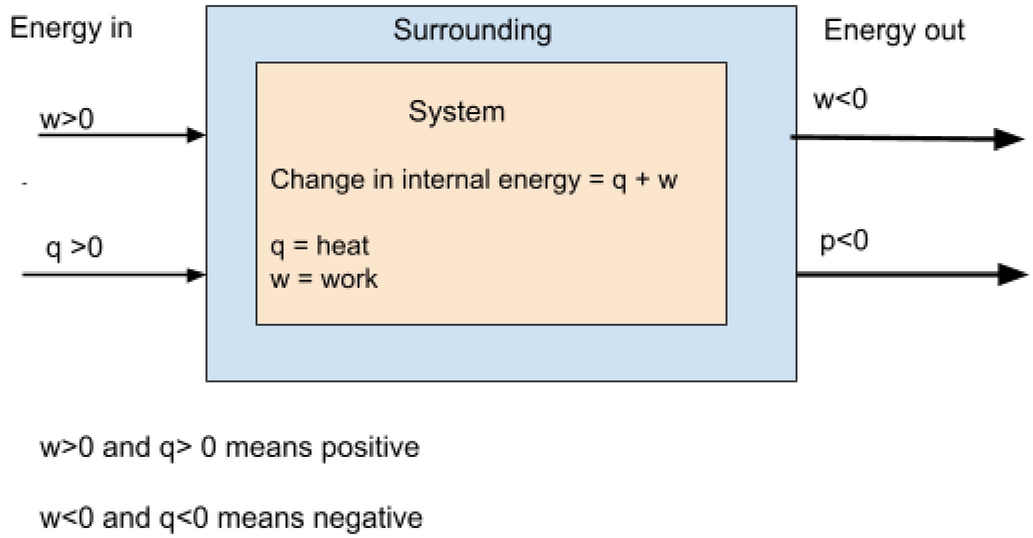
There are 2 aspects we have to be clear and that is the surrounding and the system.

$$\Delta E\ (universe) = \Delta E\ (system) + \Delta E\ (surrounding)$$

And according to the conservation of energy we know that
$\Delta E\ (universe) = 0$

Thus :
$$0 = \Delta E\ (system) + \Delta E\ (surrounding)$$
$$-\Delta E\ (surrounding) = \Delta E\ (system)$$

Sign conventions in chemical Thermodynamics :



According to the first law of thermodynamics, heat and work are the only means by which a system interchanges energy with the surrounding.

$\Delta U\ (system)\ =\ q\ +\ w$

Here, q which is the heat is microscopic while w which is the work is a macroscopic component. So in other terms, the first law of thermodynamics shows that the addition or removal of microscopic energy and the addition or removal of macroscopic energy are how the system exchanges energy with surrounding.

Now, if we were to apply the First Law of Thermodynamics to keep the internal energy constant,

$\Delta U\ (system)\ =\ q\ +\ w$, we know that the change in the internal energy should be 0.

$0\ =\ q\ +\ w$

so :

$q\ =\ -\ w$ (opposite sign)

1) Heating the system while the system does work on the surrounding:

   When we heat the system, heat is going heat to the system, so q>0 (positive) . Since work is done on the surrounding by the system, the work is going out so it becomes w< 0 (negative) according to the sign convections in thermodynamics. Thus it is possible to keep the internal energy constant in this way

2) Cooling the system while the surrounding does work on the system:

   When we cool the system, the q becomes negative (q<0) and as the surrounding does work on the system the work becomes positive, w>0, as work comes into the system from the surrounding. This is another way to keep the internal energy constant.

We can represent Work as $-p\Delta V$ where,

P is the pressure and ΔV is the change in volume

1) Is it possible to burn gasoline to release energy but do no work if external pressure is constant?

   $\Delta U\ (system) = q + w$ (First law of thermodynamics)

   $\Delta U\ (system) = q - p\Delta V$ (P is a constant)

   So the answer is NO.

2) Is it possible to burn gasoline to release energy but do no work if external temperature is constant?

   $\Delta U\ (system) = q + w$, when temperature constant the change in internal energy becomes zero.

   $0 = q + w$

$q = -w$

So the answer is NO

3) Is it possible to burn gasoline to release energy but do no work if the volume is constant?

$\Delta U\ (system) = q + w$

When volume is constant the work, w, becomes zero.

$\Delta U\ (system) = q + 0$

So the answer is YES

Enthalpy

$\Delta U\ (system) = q + w$, since w is also equal to $-\ P\Delta V$

The new thermodynamic Variable :

H = U + pV

$\Delta H = \Delta U + \Delta pV$

$\Delta U = \Delta H - \Delta pV$

$\Delta U\ (system) = q + -p\Delta V$

$\Delta H - \Delta pV = q + -p\Delta V$

$\Delta H = q + \Delta pV - p\Delta V$

So $\Delta H\ (system) = q$ at constant pressure.

Enthalpy shows the energy released from a chemical reaction at constant pressure and enthalpy is very important because almost all chemical reactions occur at constant pressure.

Standard enthalpy of formation

Enthalpy change occurs when one mole of a substance is formed from its constituent elements in the standard state. (ΔH_f^o)

A pure element in its standard state has a standard enthalpy of formation of zero. $\left[O_2(g)\ ,\ Na\ (s),\ N_2(g)\right]$

$$\Delta H^\circ_{rxn} = \Sigma\left(\Delta H^\circ_{f,[1]}\right) - \Sigma\left(\Delta H^\circ_{f,[2]}\right)$$

[1] is products
[2] is reactants
(Overall change in enthalpy of the chemical reaction)

$9C_{(s)} + 10H_{2\,(g)} \longrightarrow C_9H_{20\,(l)}$

COMPOUND	ΔHc° ($kJmol^{-1}$)
$C_{(s)}$	-394
$H_{2\,(g)}$	-286
$C_9H_{20\,(l)}$	-6125

Products : – 6125

Reactants : (9 x -394) + (10 x -286) = – 6406

Enthalpy change = – 6125 – (– 6406) = 281 KJ/mol

ΔH_{fusion} (*enthalpy of fusion*) : *energy required to turn a solid to water*

$\Delta H_{evaporation}$ (*enthalpy of evaporation*) : *energy required to turn a water to a gas*

The thermal energy produced in a chemical reaction at constant pressure is approximately equal to that produced at constant volume.

Thus in some cases, $\Delta U \approx \Delta H$

We can also say that:

$$\Delta U(therm) = \frac{3}{2} \times N \times k_B \times \Delta T$$

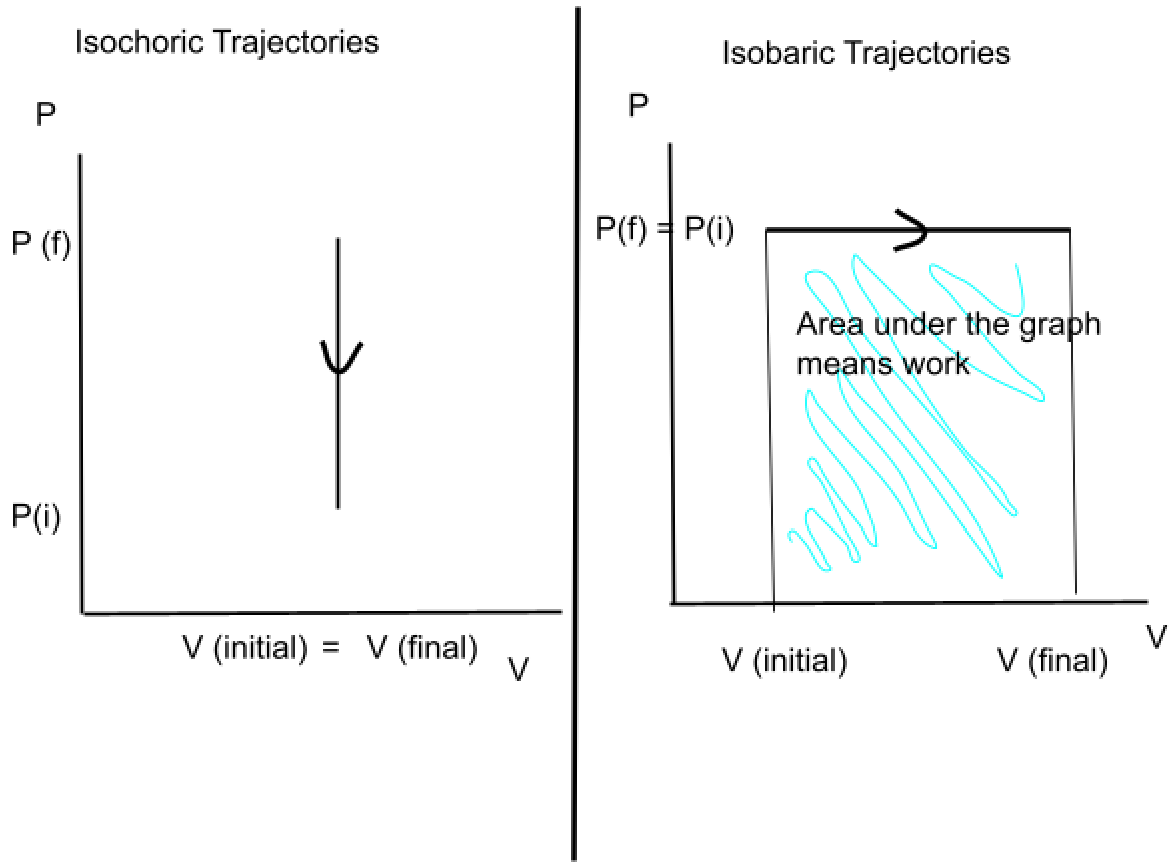
And ,

$$\Delta U = U(final) - U(initial)$$

Isochoric and Isobaric Trajectories

Isochoric Trajectories involve processes at constant volume and Isobaric Trajectories involve processes at constant pressure.

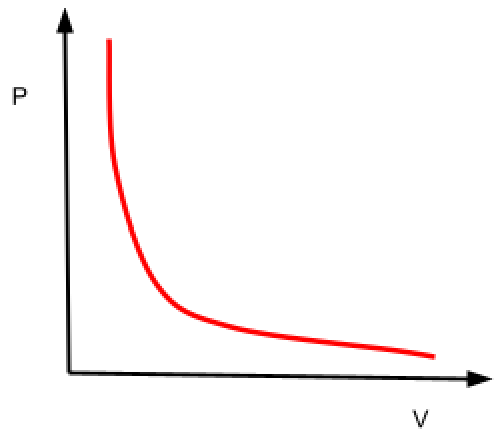
P-V (pressure - volume) coordinates for two trajectories :



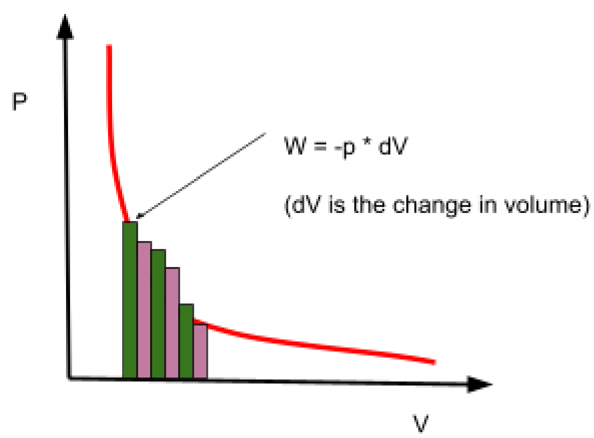
Calculating the Work done in isothermal trajectories

$W = -p\Delta V$ (P is a constant)

When we consider an isothermal trajectory the pressure is dropping as shown below:



So to calculate the work done we can divide the graph to segments as shown below:



We can say that :

$$W = -\sum_{m=1}^{n} (P_m \times \Delta V_m)$$

$$W = \lim_{\Delta V \to 0} \left[-\sum_{m=1}^{n} (P_v \times \Delta V) \right]$$

$$W = -\int_{Vi}^{Vf} PdV$$ (integral from initial volume to final volume)

Since we are dealing with perfect gases we can use the equation of $PV = nRT$

$$W = -\int_{Vi}^{Vf} nRT\frac{dV}{V}$$

$$W = -nRT\int_{Vi}^{Vf} \frac{dV}{V}$$ (since this is an isothermal process we can take nRT out)

$$W = -nRT\ ln\left(\frac{V(final)}{V(initial)}\right)$$ (equation to calculate the isothermal process)

Adiabatic Processes

This is a process in which no heat transfer is involved.

So if we compress a gas using a piston and cylinder under the adiabatic conditions where q = 0, we can determine what happens to the temperature.

As we compress the gas, we do not allow any heat exchange between the system and the surroundings so according to $\Delta U = q + w$, as q is equal to 0, we can assume that the work done is within the internal energy of the system as $\Delta U = 0 + w$.

$\Delta U = N \times \frac{3}{2} K(B)\Delta T$ (mentioned in previous chapters and N is the number of molecules)

$w = N \times \frac{3}{2} K(B)\Delta T$

So if we compress a gas and q is 0, the temperature of the gas would be greater than when q is not equal to 0.

In the next scenario, if we expand the gas and maintain q = 0, heat is not allowed to escape the system, the temperature of the gas will be less than that for which q is not equal to 0.

It is important to note how the temperature can change due to work done on a system despite no heat exchange.

Examples:

- Rapid Compression or Expansion of Gases: When a gas is quickly compressed or expanded in an insulated cylinder, such as in internal combustion engines.
- Atmospheric Processes: Certain weather phenomena, like the rise and fall of air parcels, can be modelled as adiabatic processes.

> Internal Combustion Engine

One of the most common practical applications of the adiabatic process is found in internal combustion engines. These engines rely on adiabatic compression and expansion to operate efficiently.

How It Works:

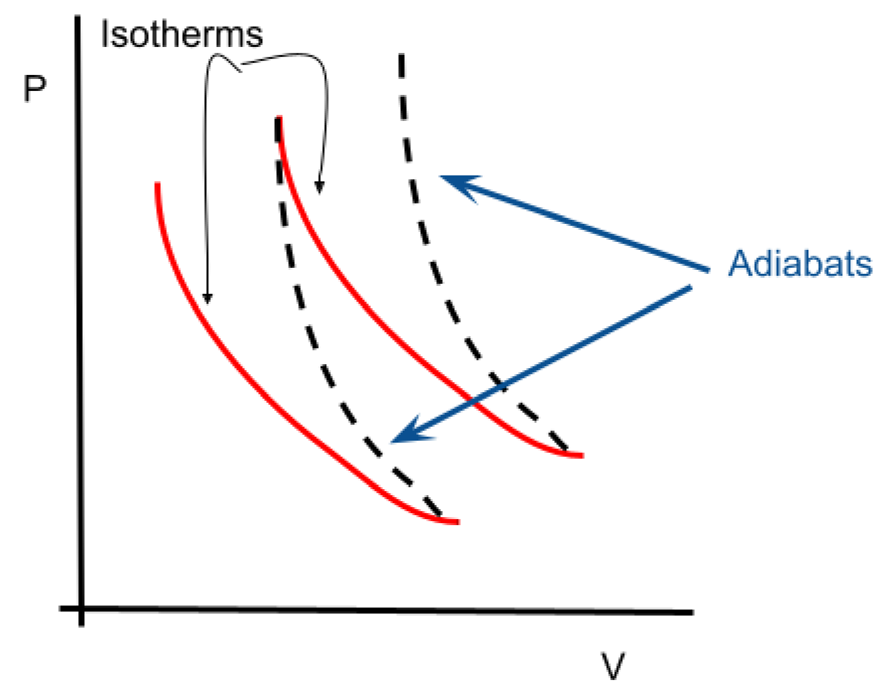
1. Intake Stroke:
   - The intake valve opens, and the piston moves down.
   - A mixture of air and fuel is drawn into the cylinder.

2. Compression Stroke:
   - The intake valve closes, and the piston moves up.
   - The air-fuel mixture is compressed adiabatically.
   - Since the cylinder walls and piston are assumed to be insulated, there is no heat transfer ($Q = 0$).
   - The work done on the gas increases its pressure and temperature significantly.

3. Power Stroke (Combustion):
   - At the end of the compression stroke, the spark plug ignites the compressed mixture.
   - The rapid combustion process can be approximated as adiabatic due to its speed.
   - The high temperature and pressure push the piston down, doing work on the piston.

4. Exhaust Stroke:
   - The exhaust valve opens, and the piston moves up again.
   - The exhaust gases are expelled from the cylinder.

Adiabatic and isothermal curves on PV graph :



Summary :

Thermodynamic Process

Isobaric process

Thermodynamic process in which the pressure remains constant is known as *isobaric process.*

Isochoric process

Thermodynamic process in which the volume remains constant is called *isochoric process.*

Adiabatic process

Thermodynamic process in which there is no heat transfer involved is called *adiabatic process.*

Isothermal process

The process in which the temperature remains constant is known as *Isothermal process.*

Internal energy

A state variable (or state function) is a property that describes the state of a system and depends only on the current state, not on how the system got there. For example, the temperature of a gas in a container is a state variable because it only depends on the current conditions of the gas, not on the process used to reach that temperature.

State variables include:

Pressure (P)

Temperature (T)

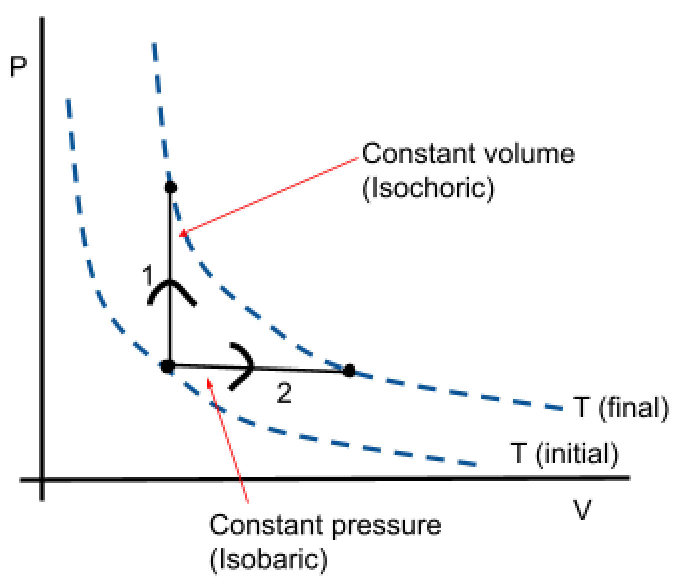
Volume (V)

The internal thermal energy of a system $\left(U_{therm}\right)$

Number of moles (n)

Number of molecules (N)

Now we will calculate the change in the internal energy of the system for isochoric (volume is unchanging) and isobaric (pressure is not changing) trajectories.



If a gas is compressible,

$\Delta U(path\ 1) = nC_v [T(final) - T(initial)]$, where C_v is the heat capacity for 1 mole of gas at constant volume (called molar specific heat capacity at constant volume).

$\Delta U(path\ 2) = nC_p [T(final) - T(initial)] - P\Delta V$, as $PV = nRT$:

$\Delta U(path\ 2) = nC_p [T(final) - T(initial)] - nRT$

So:

$\Delta U(path\ 2) = nC_p [T(final) - T(initial)] - nR [T(final) - T(initial)]$, where C_p is the heat capacity for 1 mole of gas at constant pressure (called molar specific heat capacity at constant pressure).

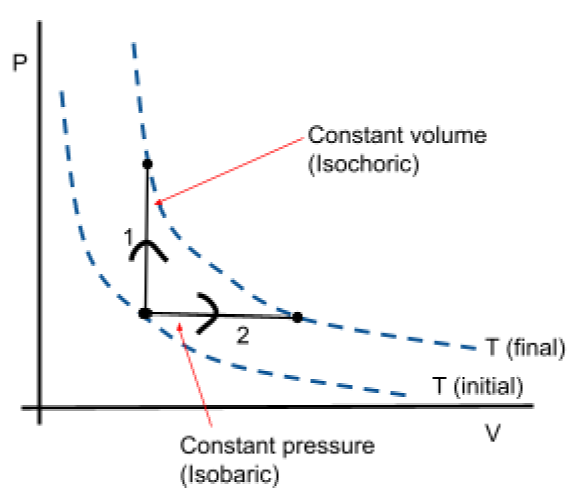
$\Delta U(path\ 1)$ is the minimum energy to get from T(initial) to T(final) and U is a state variable. So if we were to move from the initial temperature to the final temperature, then, the change in internal energy is the same whether it is isobaric or isochoric. Thus we can equate the two expressions as processes 1 and 2 have the same temperature change.

$\Delta U(path\ 1) = \Delta U(path\ 2)$

$nC_v [T(final) - T(initial)] = nC_p [T(final) - T(initial)] - nR [T(final) - T(initial)]$

** $C_v = C_p - R$

If we analyze the below graph we can further understand the concepts we learnt before :



We know that the change in internal energy in the two paths are the same. However, what about the work (is w1 = w2?)

The work done is usually the area under the graph. For instance, if we take the work done under the path 2 (at constant pressure), this would give us work. But there is no area under the graph for path 1 so the work along path 1 is 0 (at constant volume)

Looking at heat, we can say that the heat is not the same. The work is not the same even though the internal energy change is the same., $\Delta U = w + q$.

Energy along the four trajectories

We have four trajectories in thermodynamics :

1) Isobaric
2) Isochoric
3) Isothermal
4) Adiabatic

In this chapter we will be forming an expression for internal energy change ΔU along these four trajectories.

- Isobaric (pressure is a constant)

  Since pressure is a constant the change in pressure is equal to 0

  $\Delta U = q + w$. (first law of thermodynamics)

  $\Delta U = q_p - P\Delta V$ (q_p is the heat at constant pressure)

  $\Delta U = nC_p\Delta T - nR\Delta T$ (where $[T(final) - T(initial)]$ is always equal to ΔT).

  The heat required at constant pressure is equal to $nC_p\Delta T$ and $P\Delta V$ is equal to $nR\Delta T$.

  $\Delta U = n\Delta T(C_p - R)$ we know that $C_v = C_p - R$:

  Therefore, $\Delta U = nC_v\Delta T$

- Isochoric (constant volume)

  Since volume is a constant the change in volume is equal to 0

  $\Delta U = q + w$. (first law of thermodynamics)

  $\Delta U = q_v + 0$ (q_v is the heat at constant volume)

The work is equal to 0 in this case because the change in volume is equal to 0 suggesting

That there is no mechanical change in the trajectory.

$\Delta U = nC_v \Delta T$ (as $\Delta U = q_v$)

- Isothermal (Temperature is a constant)

Since temperature is a constant the change in temperature is equal to 0.

$\Delta U = N \times \frac{3}{2} K(B) \Delta T$ (ΔT is equal to 0)

$\Delta U = 0$

$\Delta U = q + w$. (first law of thermodynamics and ΔU is equal to 0)

$q = - w$

- Adiabatic (heat exchange between the surrounding and the system is equal to 0)

$\Delta U = q + w$ (q = 0)

$\Delta U = w$ (w is equal to $nC_v \Delta T$)

$\Delta U = nC_v \Delta T$

Enthalpy change along the four trajectories

In this chapter we will be forming an expression for enthalpy change ΔU along these four trajectories.

- Isobaric (pressure is a constant)

  $\Delta H = \Delta U + \Delta(PV)$ (expression for enthalpy change)

  $\Delta H = \Delta U + nR\Delta T$

  According to the previous calculations, we found out that $\Delta U = nC_p\Delta T - nR\Delta T$ (where $[T(final) - T(initial)]$ is always equal to ΔT) as the heat required at constant pressure is equal to $nC_p\Delta T$.

  Substituting this gives:

  $\Delta H = nC_p\Delta T - nR\Delta T + nR\Delta T$

  **$\Delta H = nC_p\Delta T$

- Isochoric (Volume is constant)

  $\Delta H = \Delta U + \Delta(PV)$ (expression for enthalpy change)

  $\Delta H = \Delta U + nR\Delta T$

  According to the previous calculations,

  we found out $\Delta U = nC_v\Delta T$ (as $\Delta U = q_v$)

  $\Delta H = nC_v\Delta T + nR\Delta T$

  we know that $C_v = C_p - R$

$\Delta H = n\,\Delta T(C_p - R) + nR\Delta T$

$\Delta H = nC_p\Delta T - nR\Delta T + nR\Delta T$

**$\Delta H = nC_p\Delta T$

- Isothermal (constant temperature)

  $\Delta H = \Delta U + \Delta(PV)$ (expression for enthalpy change)

  From previous calculations, we know that

  $\Delta U = N \times \frac{3}{2}K(B)\Delta T$ (ΔT is equal to 0)

  $\Delta U = 0$

  Thus :

  $\Delta H = 0 + \Delta(PV)$ (expression for enthalpy change)

  $\Delta H = nR\Delta T$

  Delta T is 0 so **$\Delta H = 0$

- Adiabatic (heat exchange between the surrounding and the system is equal to 0)

  $\Delta H = \Delta U + \Delta(PV)$ (expression for enthalpy change)

  Previous calculations:

  $\Delta U = q + w$ (q = 0)

  $\Delta U = w$ (w is equal to $nC_v\Delta T$)

  $\Delta U = nC_v\Delta T$

Thus :

$\Delta H = nC_v \Delta T + \Delta(PV)$ (expression for enthalpy change)

$\Delta H = nC_v \Delta T + nR\Delta T$

we know that $C_v = C_p - R$

$\Delta H = n\,\Delta T(C_p - R) + nR\Delta T$

$\Delta H = nC_p\Delta T - nR\Delta T + nR\Delta T$

**$\Delta H = nC_p\Delta T$

Work along the four trajectories

In this chapter we will be forming an expression for work, w, along these four trajectories.

- Isobaric (constant pressure)

  $\Delta U = q + w$ (first law of thermodynamics)

  $w = -P\Delta V$

  $w = -nR\Delta T$ ($P\Delta V = nR\Delta T$)

- Isochoric (volume is constant)

  If the volume is a constant, there is no work done (w=0), because we know that work is force times the distance and no distance means there is no work done (no macroscopic work done)

  $w = 0$

- Isothermal (Temperature is constant)

  $W = -nRT\, ln\left(\frac{V(final)}{V(initial)}\right)$ (equation to calculate the isothermal process)

  We have already calculated this.

- Adiabatic (heat exchange between the surrounding and the system is equal to 0)

  $\Delta U = q + w$ (q = 0)

  $\Delta U = w$

  $w = nC_v \Delta T$

Heat along the four trajectories

In this chapter we will be forming an expression for heat, q, along these four trajectories.

- Isobaric (pressure is constant)

$\Delta U = q + w$ (first law of thermodynamics)

$$\Delta U = q_p - \Delta(PV)$$

$$q_p = nC_p \Delta T$$

- Isochoric (constant volume)

$$q_v = nC_v \Delta T$$

- Isothermal (Temperature is constant)

$\Delta U = q + w$ (first law of thermodynamics)

$$q = -w$$

$$q = nRT\, ln\left(\frac{V(final)}{V(initial)}\right)$$

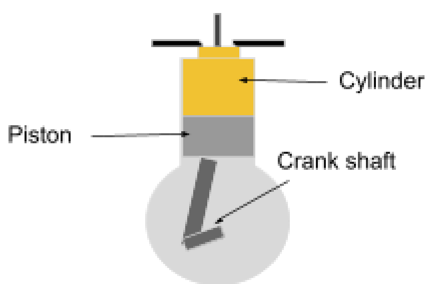
- Adiabatic (heat exchange between the surrounding and the system is equal to 0)

So q = 0.

Carnot Cycle

The Carnot cycle was created in the mid-nineteenth century by the French scientist Sadi Carnot. In this chapter, we will be dealing with the efficiency of heat engines and also the Carnot cycle.

Gasoline engine :

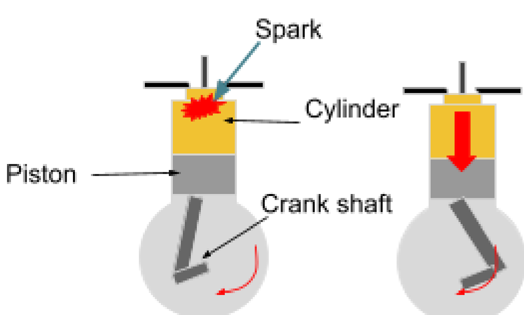


1) The Compression Stroke Explained

We start with both valves closed so nothing can escape from the cylinder.

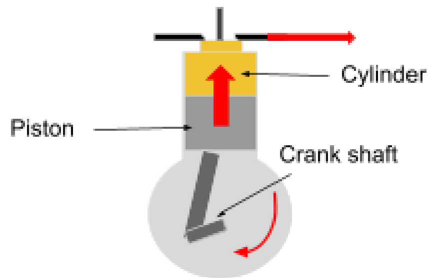
The piston starts at the bottom of the cylinder. As the crankshaft rotates, it pushes the piston up. The piston moving up compresses the mixture of gasoline and air inside the cylinder. This compressed mixture is then ready to be ignited. When ignited, it creates a force that pushes the piston back down. This up-and-down movement of the piston, powered by the crankshaft, generates the engine's power and makes the machine work.

2) Ignition and Power Stroke



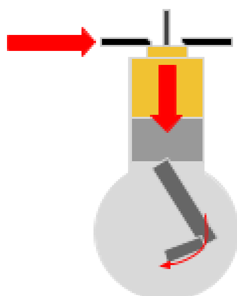
When the piston reaches the top, a spark ignites the compressed gasoline and air mixture. The spark causes the gasoline to explode, reacting with the oxygen. This explosion raises the temperature and pressure inside the cylinder. The high pressure pushes the piston back down. The piston moving down pushes the connecting rod, which turns the crankshaft. This movement generates power to keep the engine running.

3) The Exhaust Stroke Explained



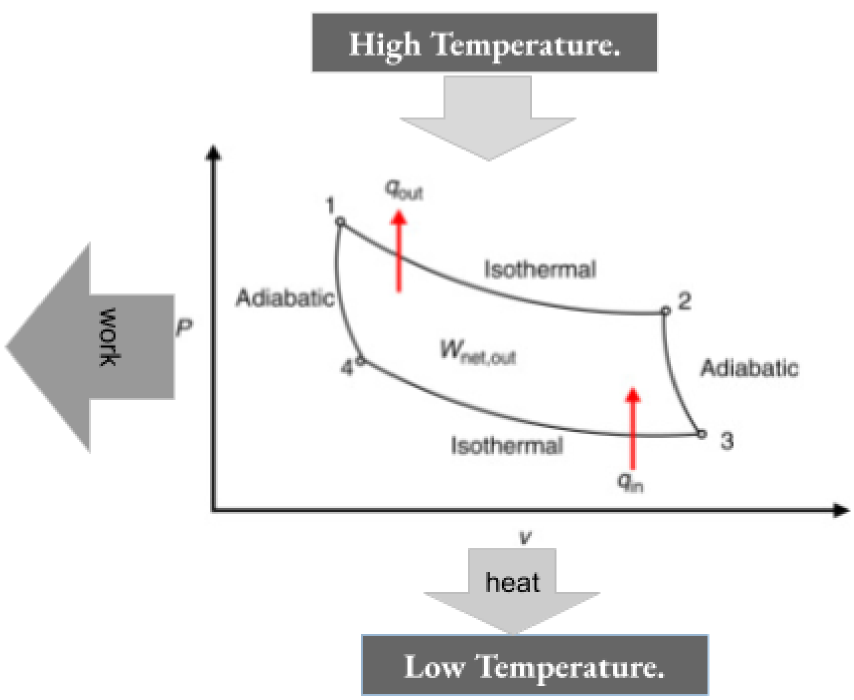
The next step is the exhaust stroke. In a four-cycle engine, there are two cycles for each spark plug ignition. The piston moves up again, but this time an exhaust valve opens. The burned gas mixture, including carbon dioxide, nitrogen oxides, and other combustion products, is pushed out. These exhaust gases are released into the exhaust system.

4) The Intake Stroke Explained



After the exhaust stroke, the piston moves down again. As it moves down, an intake valve opens. This allows a fresh mix of air and gasoline to be drawn into the cylinder. The engine is now ready to start the cycle again.

The Carnot cycle represents this cycle in a pressure-volume coordinate.



From 1 to 2 in the cycle, the pressure decreases which is comparable to the piston moving down as heat is put into the system. The gasoline has combusted with the oxygen mixture to produce a rapid increase in temperature.

2 to 3 shows the adiabatic leg of the cycle. The piston has reached the bottom of the stroke. As the crankshaft turns the volume decreases again along an isothermal trajectory and that is connected with the initial point with an adiabatic leg.

The Carnot cycle absorbs heat at a high temperature and releases heat at a low temperature.

Work done is the area under the curve.

$$\text{Efficiency of the Carnot cycle} = \frac{What\ you\ get}{What\ you\ pay\ for} = \frac{Work\ (out)}{Heat\ (in)}$$

1) Carnot cycle allows us to analyze how heat is converted to work.

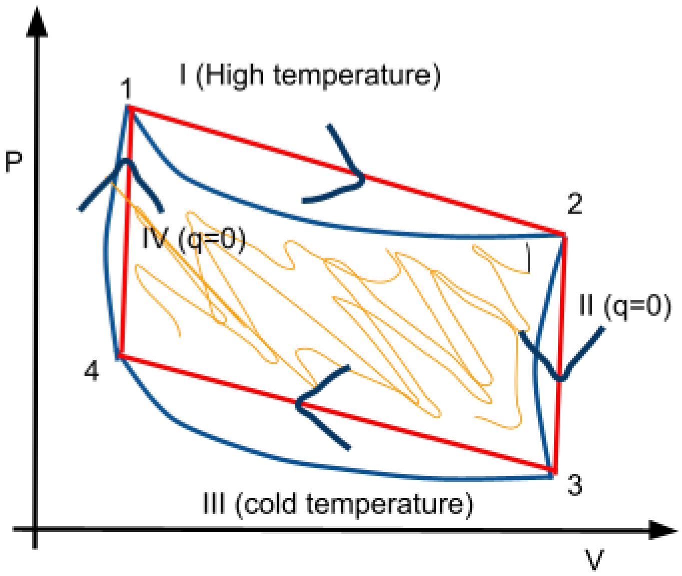
2) It tells us how disorganized forms of energy are converted to organised forms of energy.

3) Carnot cycle helps us to visualize the idea of a heat pump (used in refrigerators) where we take warm air in and expel it to cool the system down.

4) Carnot cycle is the foundation of the second law of Thermodynamics

5) It also helps us to link thermodynamics with the Stefan-Boltzmann law.

6) The Carnot cycle allows us to understand the climate system.



I : isothermal

II : adiabatic

III : isothermal

IV : adiabatic

The net work done is the area inside those four trajectories on the PV plot, so we need to find the area under each leg.

Efficiency of the Carnot cycle $= \frac{-[W(I) + W(II) + W(III) + W(IV)]}{q_{\ in}} = \frac{Work\ (out)}{Heat\ (in)}$

** The minus sign is there because the efficiency usually is positive but in the case of a heat engine work is done on the surrounding. So when we add up the work term in each of those legs, we get a negative number. Multiplying this negative number with a negative one gives us a positive answer.

Leg I (isothermal) :

$\Delta U \ = \ q \ + \ w = 0$

$q \ = \ - \ w(I)$

$W(I) \ = \ - \ nRT_{\ high}\ ln\left(\frac{V(final)}{V(initial)}\right)$, *temperature in this case is hot.*

** $W(I) \ = \ - \ nRT_{\ high}\ ln\left(\frac{V(2)}{V(1)}\right)$, *temperature in this case is hot.*

Leg II (adiabatic) , q = 0 :

$$\Delta U = q + w(II)$$

$$\Delta U = w(II)$$

$$\Delta U = nC_v(T_f - T_i)$$

** $w(II) = nC_v(T_{cold} - T_{high})$

Leg III (isothermal) :

** $W(III) = -nRT_{cold}\, ln\left(\frac{V(4)}{V(3)}\right)$, *temperature in this case is cold.*

Leg IV (adiabatic) :

$$\Delta U = q + w(IV)$$

** $w(IV) = nC_v(T_{high} - T_{cold})$

Efficiency = $\frac{-[-nRT_{high}\, ln\left(\frac{V(2)}{V(1)}\right) + nC_v(T_{cold} - T_{high}) - nRT_{cold}\, ln\left(\frac{V(4)}{V(3)}\right) + nC_v(T_{high} - T_{cold})]}{q_{in}}$

We know that we put the heat at a high temperature into the system. That's the heat generated when gasoline is added to each of these cylinders and explodes to drive the piston down.

We are adding q_{in} into the hot side through an isothermal trajectory.

$$\Delta U = q + w$$

$$q = -w$$

$$q_{in} = -w(I)$$

$$q_{in} = nRT_{high} \, ln\left(\frac{V(2)}{V(1)}\right)$$

$$\text{Efficiency} = \frac{-[-nRT_{high} \, ln\left(\frac{V(2)}{V(1)}\right) + nC_v(T_{cold} - T_{high}) - nRT_{cold} \, ln\left(\frac{V(4)}{V(3)}\right) + nC_v(T_{high} - T_{cold})]}{nRT_{high} \, ln\left(\frac{V(2)}{V(1)}\right)}$$

$$\text{Efficiency} = \frac{-[-nRT_{high} \, ln\left(\frac{V(2)}{V(1)}\right) + nC_v(T_{cold} - T_{high}) - nRT_{cold} \, ln\left(\frac{V(4)}{V(3)}\right) + nC_v(T_{high} - T_{cold})]}{nRT_{high} \, ln\left(\frac{V(2)}{V(1)}\right)}$$

$$\text{Efficiency} = \frac{-[-nRT_{high} \, ln\left(\frac{V(2)}{V(1)}\right) - nRT_{cold} \, ln\left(\frac{V(4)}{V(3)}\right)]}{nRT_{high} \, ln\left(\frac{V(2)}{V(1)}\right)}$$

$$\text{Efficiency} = \frac{nRT_{high} \, ln\left(\frac{V(2)}{V(1)}\right) + nRT_{cold} \, ln\left(\frac{V(4)}{V(3)}\right)]}{nRT_{high} \, ln\left(\frac{V(2)}{V(1)}\right)}$$

$$\text{Efficiency} = \frac{nRT_{high} \, ln\left(\frac{V(2)}{V(1)}\right) + nRT_{cold} \, ln\left(\frac{V(4)}{V(3)}\right)]}{nRT_{high} \, ln\left(\frac{V(2)}{V(1)}\right)}$$

$$\text{Efficiency} = \frac{T_{high} \, ln\left(\frac{V(2)}{V(1)}\right) + T_{cold} \, ln\left(\frac{V(4)}{V(3)}\right)]}{T_{high} \, ln\left(\frac{V(2)}{V(1)}\right)}$$

We will find a relationship that allows us to ratio temperature to volume :

We can convert the expression of $\Delta U = q + w$:

$dU = dq + dw$, dq is 0.

$dU = dw$

$$dW = nC_v dT$$

$$dU = -PdV = -nRT\frac{dV}{V}$$

$$-nRT\frac{dV}{V} = nC_v dT$$

$$-nRT\frac{dV}{V} = nC_v \frac{dT}{T}$$

$$nC_v \int_{T(Initial)}^{T(final)} \frac{dT}{T} = -nR \int_{V(Initial)}^{V(final)} \frac{dV}{V}$$

** $nC_v ln\left(\frac{T(final)}{T(initial)}\right) = -nRln\left(\frac{V(final)}{V(initial)}\right)$

Further simplifying the Carnot cycle efficiency equation with the above found relationship :

Leg II (adiabatic leg) :

$$nC_v ln\left(\frac{T(final)}{T(initial)}\right) = -nRln\left(\frac{V(final)}{V(initial)}\right)$$

Further simplifying the Carnot cycle efficiency equation with the above found relationship
Applying this :

$$nC_v ln\left(\frac{T(cold)}{T(high)}\right) = -nRln\left(\frac{V(3)}{V(2)}\right)$$

We can invert the sign by flipping the ratio in the natural log:

$$-nC_v ln\left(\frac{T(high)}{T(cold)}\right) = -nRln\left(\frac{V(3)}{V(2)}\right)$$

Leg IV :

$$nC_v ln\left(\frac{T(high)}{T(cold)}\right) = -nRln\left(\frac{V(1)}{V(4)}\right)$$

We can invert the sign by flipping the ratio in the natural log:

$$nC_v ln\left(\frac{T(high)}{T(cold)}\right) = nRln\left(\frac{V(4)}{V(1)}\right)$$

So :

$$-nC_v ln\left(\frac{T(high)}{T(cold)}\right) = -nRln\left(\frac{V(3)}{V(2)}\right)$$

$$nC_v ln\left(\frac{T(high)}{T(cold)}\right) = nRln\left(\frac{V(3)}{V(2)}\right)$$

Adding the above to the equation in Leg IV gives :

$$nRln\left(\frac{V(4)}{V(1)}\right) = nRln\left(\frac{V(3)}{V(2)}\right)$$

$$\cancel{nRln}\left(\frac{V(4)}{V(1)}\right) = \cancel{nRln}\left(\frac{V(3)}{V(2)}\right)$$

$$\left(\frac{V(4)}{V(1)}\right) = \left(\frac{V(3)}{V(2)}\right)$$

$$\text{Efficiency} = \frac{T_{high}\, ln\left(\frac{V(2)}{V(1)}\right) + T_{cold}\, ln\left(\frac{V(4)}{V(3)}\right)]}{T_{high}\, ln\left(\frac{V(2)}{V(1)}\right)}$$

If we rearrange, $\left(\frac{V(4)}{V(1)}\right) = \left(\frac{V(3)}{V(2)}\right)$, to a format of $\frac{V(4)}{V(3)}$ by cross multiplying we get :

$$\frac{V(4)}{V(3)} = \frac{V(1)}{V(2)}$$

$$\frac{V(2)}{V(1)}$$

$$\text{Efficiency} = \frac{T_{high}\, ln\left(\frac{V(2)}{V(1)}\right) + T_{cold}\, ln\left(\frac{V(4)}{V(3)}\right)\,]}{T_{high}\, ln\left(\frac{V(2)}{V(1)}\right)}$$

$$\text{Efficiency} = \frac{T_{high}\, ln\left(\frac{V(2)}{V(1)}\right) + T_{cold}\, ln\left(\frac{V(1)}{V(2)}\right)\,]}{T_{high}\, ln\left(\frac{V(2)}{V(1)}\right)}$$

We can invert the sign by flipping the ratio in the natural log:

$$\text{Efficiency} = \frac{T_{high}\, ln\left(\frac{V(2)}{V(1)}\right) - T_{cold}\, ln\left(\frac{V(2)}{V(1)}\right)\,]}{T_{high}\, ln\left(\frac{V(2)}{V(1)}\right)}$$

$$\text{Efficiency} = \frac{T_{high}\, ln\left(\frac{V(2)}{V(1)}\right) - T_{cold}\, ln\left(\frac{V(2)}{V(1)}\right)\,]}{T_{high}\, ln\left(\frac{V(2)}{V(1)}\right)}$$

$$** \text{ Efficiency} = \frac{T_{high} - T_{cold}}{T_{high}}$$

The temperature should be in Kelvin. This tells you how to increase the temperature intrinsic to the high-temperature side of the cycle and low temperature to be as low as possible to get the greatest difference in the number. A condenser was added to the exhaust steam engine to drop its temperature and the efficiency went up.

In a refrigerator, the heat is extracted from the inside and released out to the surrounding. This way, we put work in to pump heat from the cold side.

Both the refrigerator and the heat pump have the same work and heat flow paths. They use work to transfer heat from a colder to a hotter object. The key difference is that the refrigerator focuses on cooling the colder object while the heat pump focuses on warming the hotter object.

Heat pumps are essential because they can deliver heat to houses or other buildings with a huge gain.

Let's say we burn 100 units of energy from natural gas combustion and is the power plant is 50% efficient then it will generate 50 units of electrical power. The power grid with 6 % loss would deliver 47 units of electrical energy. The heat pump is usually with coefficient of performance of 5, so 47 into 5 gives us 235 units of heat added to the house (To increase the temperature of the house).

However, if we burn 100 units of natural gas in a furnace in the basement of the house, then the heat added to the house will be 90 units as a furnace is usually 90% efficient.

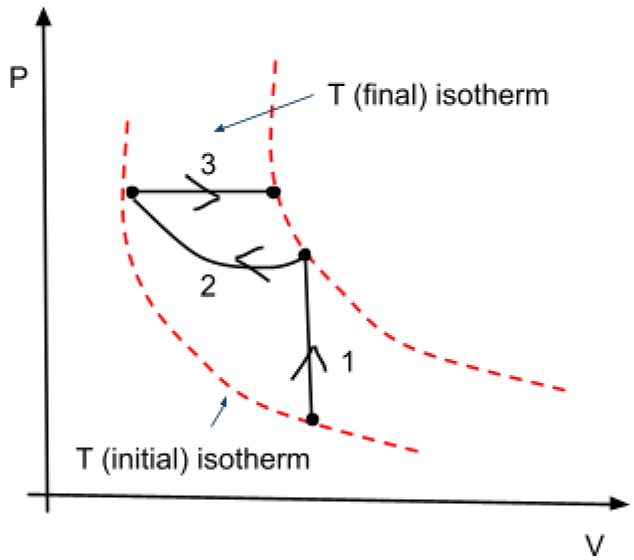
This shows that using a heat pump allows more heat to be added to the house and if we calculate the gain factor by dividing 235 by 90 then it will be 2.6.

A heat pump of a coefficient of performance of 5 (COP of 5) means that the heat pump can output 5 times as much thermal energy to the high temperature reservoir as the amount of electricity expended.

Trivia Question 2

Question 1:

Identify the processes below :



Answer 1 :

Process 1 : isochoric - volume stays the same while the pressure does not.

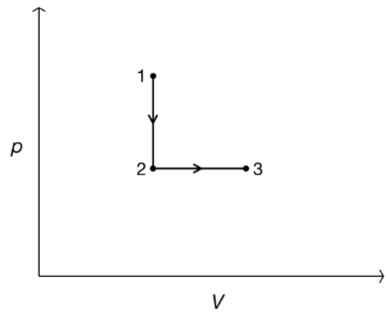
Process 2 : adiabatic - it foes from one isotherm to another

Process 3 : Isobaric - pressure stays the same while the volume does not.

Question 2 :

Consider 2.00 moles of a monoatomic ideal gas initially occupying 1.0 L at a pressure of 3 atm in a cylinder with a frictionless, movable piston. The gas undergoes a two-stage process to reach a final state of 3.0 L at 1 atm. First, in an isochoric process (constant volume), the pressure decreases from 3 atm to 1 atm with no work done, and heat is removed, calculated using $Q1 = nC_v \Delta T$ where $C_v = \frac{3}{2}R$. Second, in an isobaric process (constant pressure), the,gas expands from 1.0 L to 3.0 L at 1 atm, with heat added, calculated using.

$Q2 = nC_p\Delta T$ where $C_p = \frac{5}{2}R$. The total heat added is the sum of the heat from both stages.



Calculate the heat added during the process. [R = 0.0821 L·atm/mol·K]

Answer 2 :

$q12 = nC_v\Delta T = nC_v(T_2 - T_1)$ where $C_v = \frac{3}{2}R$.

$q23 = nC_p\Delta T = nC_p(T_3 - T_2)$ where $C_p = \frac{5}{2}R$.

$q(total) = q12 + q23$

$q(total) = nC_v(T_2 - T_1) + nC_p(T_3 - T_2)$

$q(total) = nC_v(T_2 - T_1) - nC_p(T_2 - T_3)$

$q(total) = nC_v(T_2 - T_1) - nC_p(T_2 - T_1)$, $since\ T_1 = T_3$

$q(total) = -n\left[(-C_v + C_p)(T_2 - T_1)\right]$

$q(total) = -n\left[(-\frac{3}{2}R + \frac{5}{2}R)(T_2 - T_1)\right]$

$q(total) = -nR(T_2 - T_1)$

$q(total) = -nRT_2 + nRT_1$

$q(total) = -P_2V_2 + P_1V_1$

$q(total) = -(1.0L)(1atm) + (1.0L)(3atm) = (2.0atm) \times L \times \frac{101J}{1atm\times L} = 202J$

Question 3:

How does a gasoline engine work ?

Answer 3:

The Four-Stroke Cycle of a Gasoline Engine

A gasoline engine operates through a four-stroke cycle involving a crankshaft, connecting rod, piston, spark plug, and valves. Here's a concise overview of each stroke:

1. Intake Stroke: The cycle begins with the piston at its lowest point. As the crankshaft rotates, it pushes the connecting rod, which moves the piston upwards. During this stroke, the intake valve opens, allowing a mixture of gasoline vapour and oxygen to enter the cylinder.

2. Compression Stroke: As the piston continues to move upwards, the intake valve closes. The piston compresses the fuel-oxygen mixture as it reaches the top of the stroke. This compression prepares the mixture for ignition.

3. Power Stroke: At the top of the stroke, a spark plug ignites the compressed mixture. The resulting combustion significantly increases the temperature and pressure, forcing the piston down. This downward movement, known as the power stroke, drives the connecting rod and rotates the crankshaft, producing mechanical energy.

4. Exhaust Stroke: The crankshaft's rotation pushes the connecting rod back up, moving the piston towards the top of the cylinder again. This time, the exhaust valve opens, allowing the spent gases to escape through the exhaust pipe. As the piston reaches the top, the intake valve opens, and a new charge of gasoline vapor and oxygen enters the cylinder, ready to start the cycle anew.

This four-stroke process—intake, compression, power, and exhaust—repeats continuously, enabling the engine to convert chemical energy into mechanical energy efficiently.

Question 4:

What is the relationship between the energy inputs and outputs for a thermodynamic cycle?

Answer 4:

The energy inputs and outputs are equivalent.

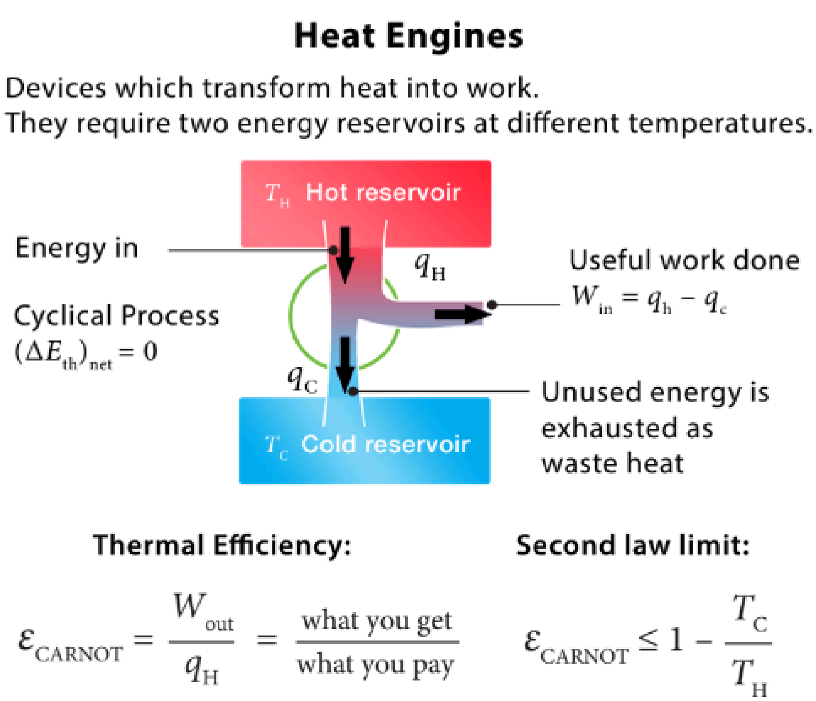
Question 5:

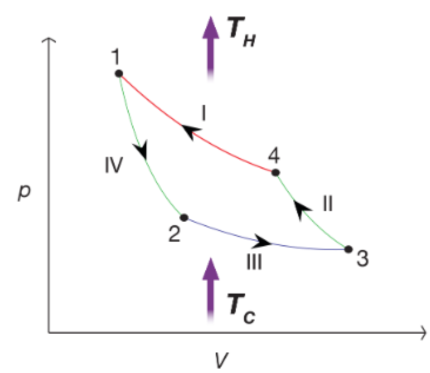
Why is it possible to achieve a higher efficiency in a theoretical Carnot cycle engine than in a real-life engine?

Answer 5:

Carnot cycle operates reversibly whereas real engines operate irreversibly.

Summary :





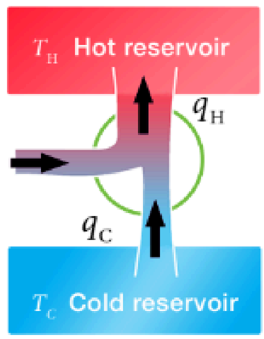
(Carnot cycle)

Heat Pumps

Devices which use work to transfer heat from a colder object to a hotter object.

Work must be done to transfer energy from cold to hot.

Heat energy is extracted from the cold reservoir.



Energy $q_H = q_C + W_{in}$ is exhausted to the hot reservoir.

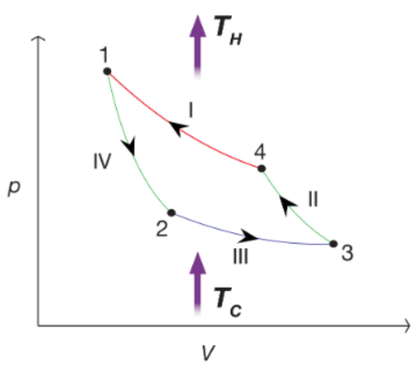
Cyclical Process
$(\Delta E_{th})_{net} = 0$

Coefficient of performance:

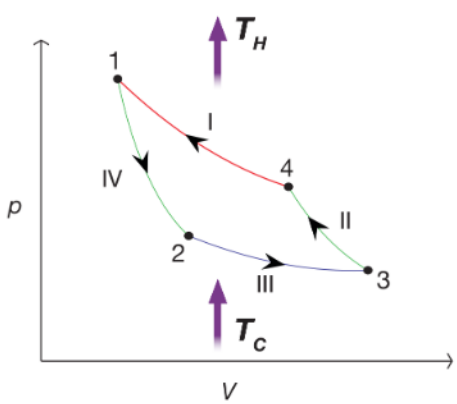
$$COP_{HP} = \frac{q_H}{W_{in}} = \frac{\text{what you get}}{\text{what you pay}}$$

Second law limit
for a heat pump that is used to heat a house:

$$COP_{HP} \leq \frac{T_H}{T_H - T_C}$$



(Carnot cycle for a refrigerator)



(Carnot cycle for an air conditioner)

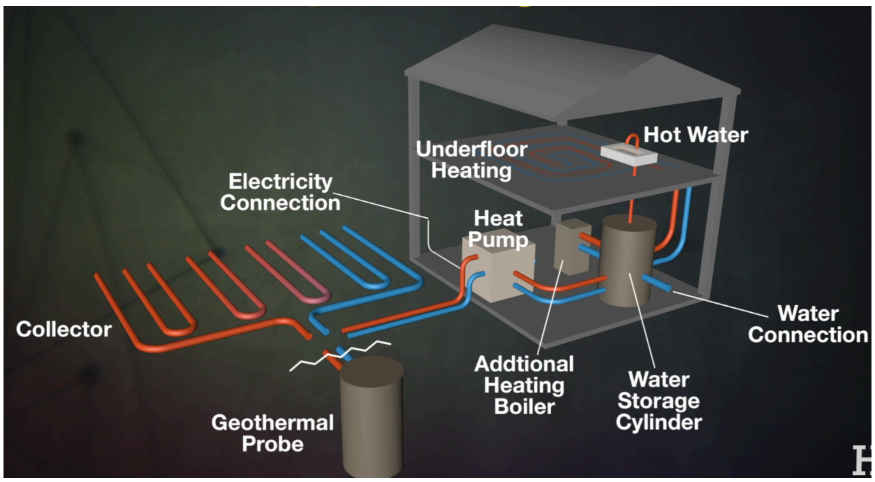
(Picture credits: Harvard PS11.1aX)

Geothermal energy.

There are 2 types of geothermal energy :

1) Low temperature geothermal
2) High temperature geothermal

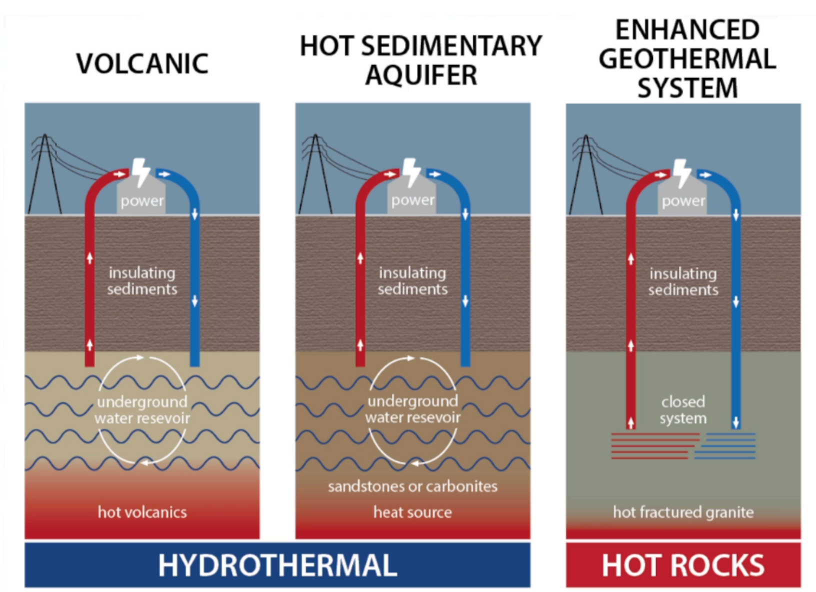
Low temperature Geothermal :



(Picture credit: Harvard PS11.1aX)

The low temperature Geothermal uses a heat exchange system in the ground and stabilizes the temperature against which heat pump can operate. The heat pump can be driven from the aire temperature outside or stablilze the temperature using the ground temperature by either heating or cooling.

High temperature Geothermal :



(Picture credit: Harvard PS11.1aX)

There are 3 types of the high temperature Geothermal :

1) Volcanic - the volcanic lava can be used to create high temperatures that is used to create steam inorder to drive the turbine to generate electricity.

2) Hot sedimertary Aquifier : Cold water is pumped to the hot water underground and extract the high temperature of the water.

3) Enhanced Geothermal system - Insert the cold water into the hot rocks below which has a temperature of 300 degree celsius. The steam formed is extracted and used to drive the turbine.

Wind energy.

Wind energy is a form of renewable energy uses the power of wind to generate electricity. It is one of the fastest-growing sources of energy in the world.

Wind energy systems convert kinetic energy from the wind into mechanical power, which can then be converted into electricity using generators.

When the wind blows, the wind blades spin which turns the rotor shaft connected to it. However, the rotor shaft revolves 18 rounds per minute which is considerably slow to generate electricity.

Therefore, the rotor shaft is connected to a set of gears which increases the rotation to about 1800 revolutions per minute. With such a high speed, the generator will be able to produce electricity.

There are two types of wind turbines :

1) Horizontal - axis wind turbines - blades rotate around a horizontal axis.
2) Vertical - axis wind turbines - blades rotate around a vertical axis.

(Vertical - axis wind turbines)

(Horizontal - axis wind turbines)

Piezoelectricity

Contents

Piezoelectricity Introduction

Piezoelectricity is a fascinating and innovative concept in the realm of materials science and engineering. It refers to the electric charge that accumulates in certain materials in response to applied mechanical stress.

It is derived from the Greek words "piezein," meaning "to press," and "electricity," piezoelectricity describes the ability of specific materials to generate an electrical charge when subjected to force or pressure.

Piezoelectricity is about the interaction between mechanical stress and electrical charge. Certain materials, known as piezoelectric materials, exhibit this unique property. When these materials are compressed, stretched, or otherwise deformed, they produce an electric charge. When an electric field is applied to them, they can change shape or deform.

Piezoelectricity comes from the internal structure of piezoelectric materials. These materials have a crystalline structure with asymmetrical charge distributions. When mechanical stress is applied, it causes a shift in these internal charge distributions, leading to the generation of an electric field. This phenomenon occurs in materials such as quartz, certain ceramics, and even some biological materials like bone.

Applications

1. Energy Harvesting: Piezoelectric materials are increasingly used in energy harvesting technologies to capture and convert mechanical vibrations into electrical energy. This has applications in powering small electronic devices and sensors.

2. Medical Devices: In the medical field, piezoelectric materials are used in ultrasound imaging, where they generate and detect sound waves to create detailed images of internal body structures.

3. Aerospace and Defense: In aerospace and defense applications, piezoelectric materials are used in vibration sensors, structural health monitoring, and active vibration control systems.

Piezoelectricity was first discovered in 1880 by two brothers and French scientists, Jacques and Pierre Curie. When they were experimenting with different crystals, they found out that when they apply some mechanical pressure to a crystal like quartz, it released electrical charge. This is now known as the piezoelectric effect.

In World War I, piezoelectricity was used for practical applications in sonar. A sonar works by connecting a voltage to a piezoelectric transmitter. This is the inverse piezoelectric effect in action, which converts electrical energy into mechanical sound waves because the pizeoelectric effect is known as the conversion of mechanical pressure to electrical charge.

The sound waves travel through the water till they hit an object. They then return back to a source receiver after being reflected. This receiver uses the direct piezoelectric effect to convert sound waves into an electrical voltage, which a signal-processing device can then process. Using the time between when the signal left and when it returned, an object's distance can easily be calculated.

In World War II advanced the technology even further as researchers from the United States, Russia, and Japan worked to craft new man-made piezoelectric materials called ferroelectrics. This research led to two man-made materials used alongside natural quartz crystal, barium titanate, and lead zirconate titanate.

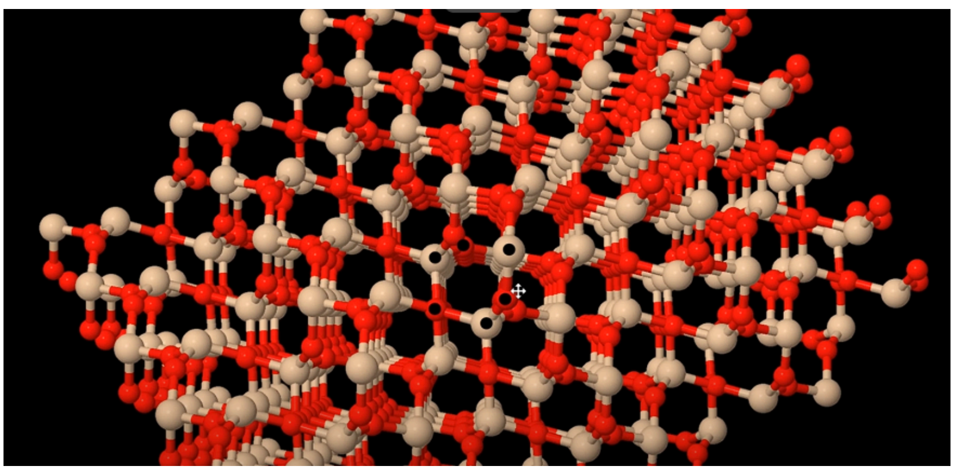
Principles

The crystal structure of piezoelectric materials is very important. These materials have a lattice arrangement that lacks a center of symmetry. This asymmetry is crucial because it allows the material to generate an electric charge in response to mechanical deformation.

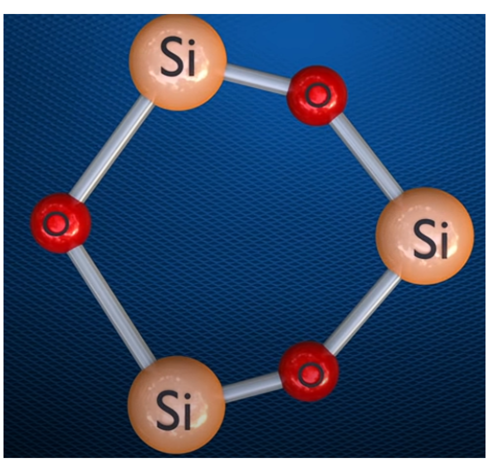
Let's take Quartz crystal for example :

(Quartz crystal)

The lattic structure of quartz contains silicon and oxygen atoms as quartz is made up of silicon dioxide.

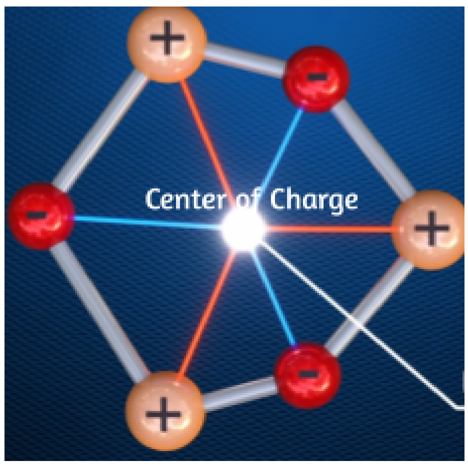


The above picture shows the specific angle for the lattic arrangement. If we zoom into the hexagonal structure we can see that we have three silicon atoms and two oxygen atoms.



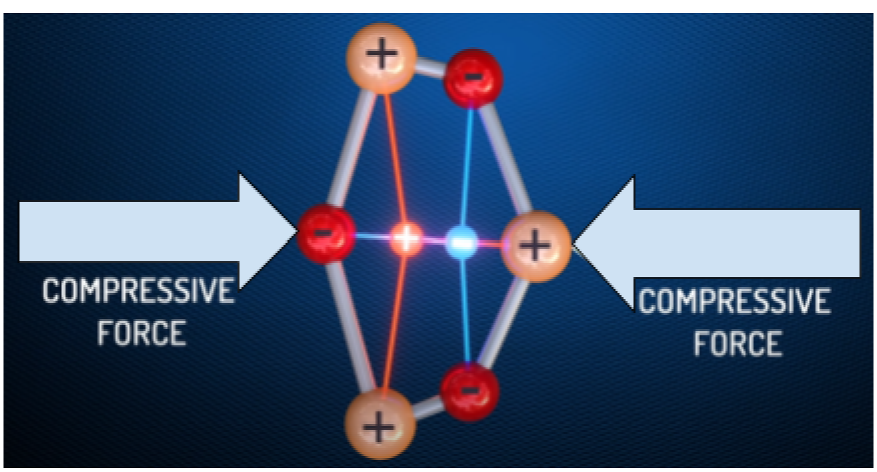
The lattic arrangement is not symmetrical.

The oxygen and silicon atoms share one electron. As the oxygen atom is small than the silicon atom, the shared electron is closer to the oxygen nucleus. Therefore, the oxygen's nucleus exerts a greater force on the shared electron. The oxygen atom is more electronegative than the silicon atom and the oxygen atom has a slightly negative charge and the silicon atom has a slightly positive charge. This is called electronegativity. They form a dipole together due to the presence of slightly negative and slightly positive charges.



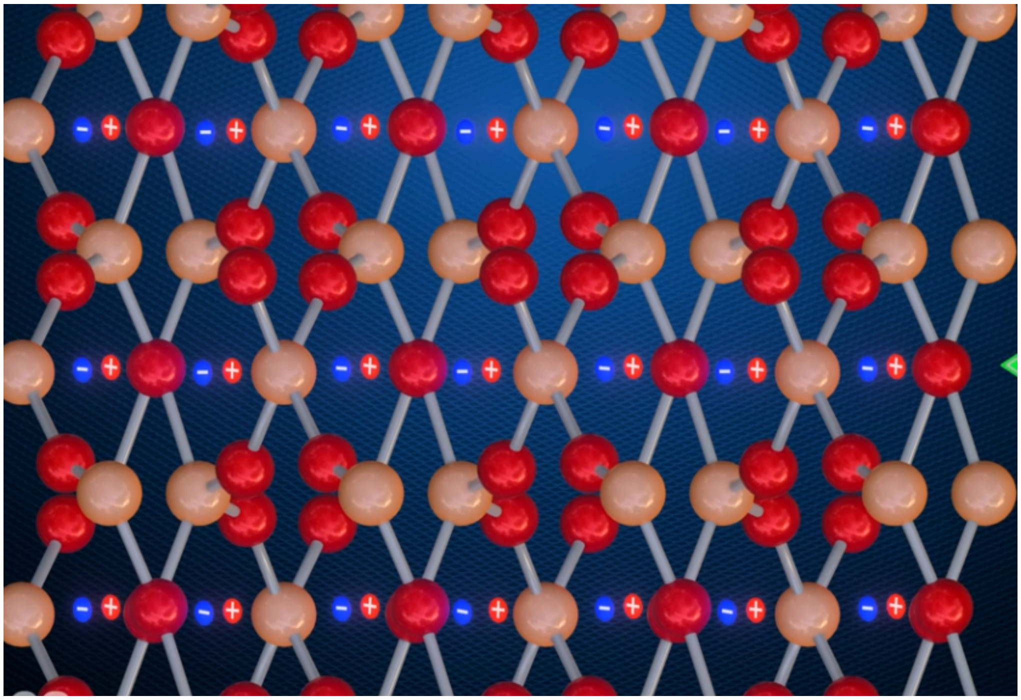
The structure has the center of charge of the positive and the negative charges coincide.

When mechanical stress is applied to such a material, it causes a shift in the distribution of electrical charges within the crystal lattice. This shift results in an electrical potential.

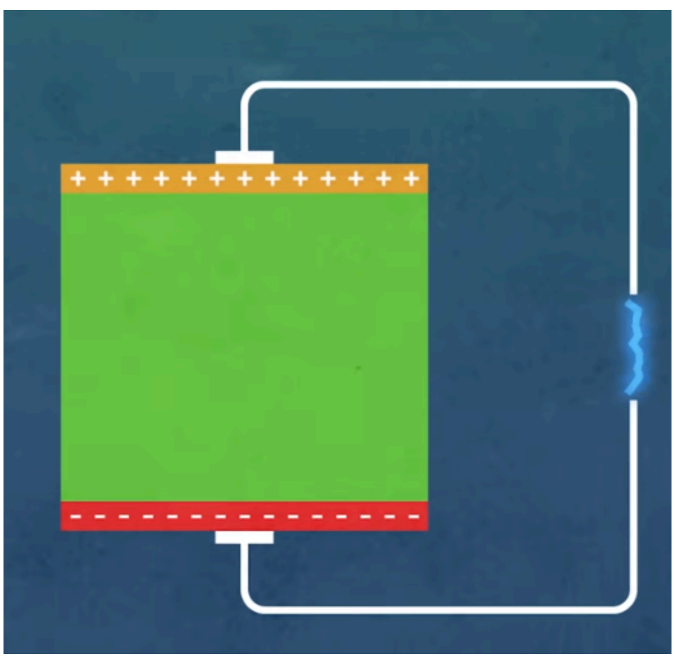


An electric field would be created between the net positive and negative charges. This leads to the generation of a small voltage across it.

When the whole lattic is compressed:



If we take the compressed lattic arrangement (specific) :



When we connect the ends of the compressed lattic using a wires which are very close together, the positive side would try to pull the electrons and the negative side would have repelling electrons. Since the wires are close together, electricity would jump through the gap.

Conversely, when an electric field is applied, the material deforms in response, demonstrating the inverse piezoelectric effect.

When a voltage is applied to the piezoelectric material, the negative and positive charges will experience forces in the opposite directions and the lattic will get stretched and when the voltage is reversed, the lattic gets compressed.

Piezoelectric materials are characterized by their ability to convert mechanical energy into electrical energy and vice versa.

This bidirectional conversion is described by two primary effects:

1) The direct piezoelectric effect
2) The inverse piezoelectric effect.

The direct piezoelectric effect occurs when mechanical stress is applied to the material, leading to the generation of electrical charges. This is used in sensors and transducers, where the material's response to external forces is converted into an electrical signal.

Inverse piezoelectric effect takes place when an electric field is applied, causing the material to undergo mechanical deformation. This effect is used in actuators and precision movement systems.

When we take diamond and squeeze it, all the carbon atoms are neutral so it won’t be classified as a Piezoelectric material.

Criteria to Be a Piezoelectric Material

To be classified as a piezoelectric material, certain criteria must be met. These criteria involes around the material's crystal structure, symmetry, and inherent properties.

1. Crystal Structure :

The crystal structure of a material is important to its piezoelectric properties. For a material to be piezoelectric, it must have a non-centrosymmetric crystal lattice. The crystal structure must lacks a center of symmetry. When it lacks a center of symmetry, it will allow for the separation of positive and negative charges when mechanical stress is applied.

2. Piezoelectric Coefficients :

Piezoelectric materials are characterized by their piezoelectric coefficients, which means their ability to convert mechanical stress into electrical charge and vice versa. Higher coefficients indicate a stronger piezoelectric response.

The selection of materials for applications usually depends on these coefficients, which determine the material's efficiency and effectiveness in converting energy.

3. Ferroelectric Properties :

Many piezoelectric materials are also ferroelectric. Ferroelectricity is a property where the material has a spontaneous electric polarization which can be reversed by an external electric field. This characteristic develops the piezoelectric effect by providing greater control over the material's polarization state.

4. Mechanical and Electrical Properties :

The effectiveness of a piezoelectric material also depends on its mechanical and electrical properties. For maximum performance, a piezoelectric material should have high mechanical stiffness and low electrical conductivity.

Mechanical stiffness ensures that the material can withstand applied stresses without significant deformation, while a low electrical conductivity minimizes energy losses and enhances the efficiency.

5. Symmetry and Orientation :

The symmetry of the crystal lattice influences the piezoelectric properties. Piezoelectric materials must have specific crystal symmetries that allow for the piezoelectric effect to manifest.

6. Temperature Stability :

The temperature stability of a piezoelectric material is essential for its operation. Materials should maintain their piezoelectric properties over a range of temperatures. Some piezoelectric materials exhibit significant changes in their properties with temperature fluctuations, so materials used in extreme conditions must have high thermal stability.

In summary, a material must meet several criteria to be classified as piezoelectric, including possessing a non-centrosymmetric crystal structure, having required piezoelectric coefficients, and demonstrating specific mechanical and electrical properties.

Additionally, ferroelectric characteristics, crystal symmetry, and temperature stability need to be taken into account.

Electronic Stethoscope

Electronic stethoscopes use electronic components to capture, amplify, and sometimes record these sounds.

1. Sound Detection

The main component of an electronic stethoscope is its sensitive acoustic sensors, often located in the chest piece or diaphragm. These sensors, usually piezoelectric, detect vibrations caused by body sounds. When a patient's body produces sounds, these vibrations create mechanical waves that the sensors convert into electrical signals.

2. Signal Amplification

The sensors capture the acoustic signals and are sent to an amplifier within the stethoscope. The amplifier boosts the strength of these electrical signals, making the body sounds more audible. The level of amplification can often be adjusted.

3. Signal Processing

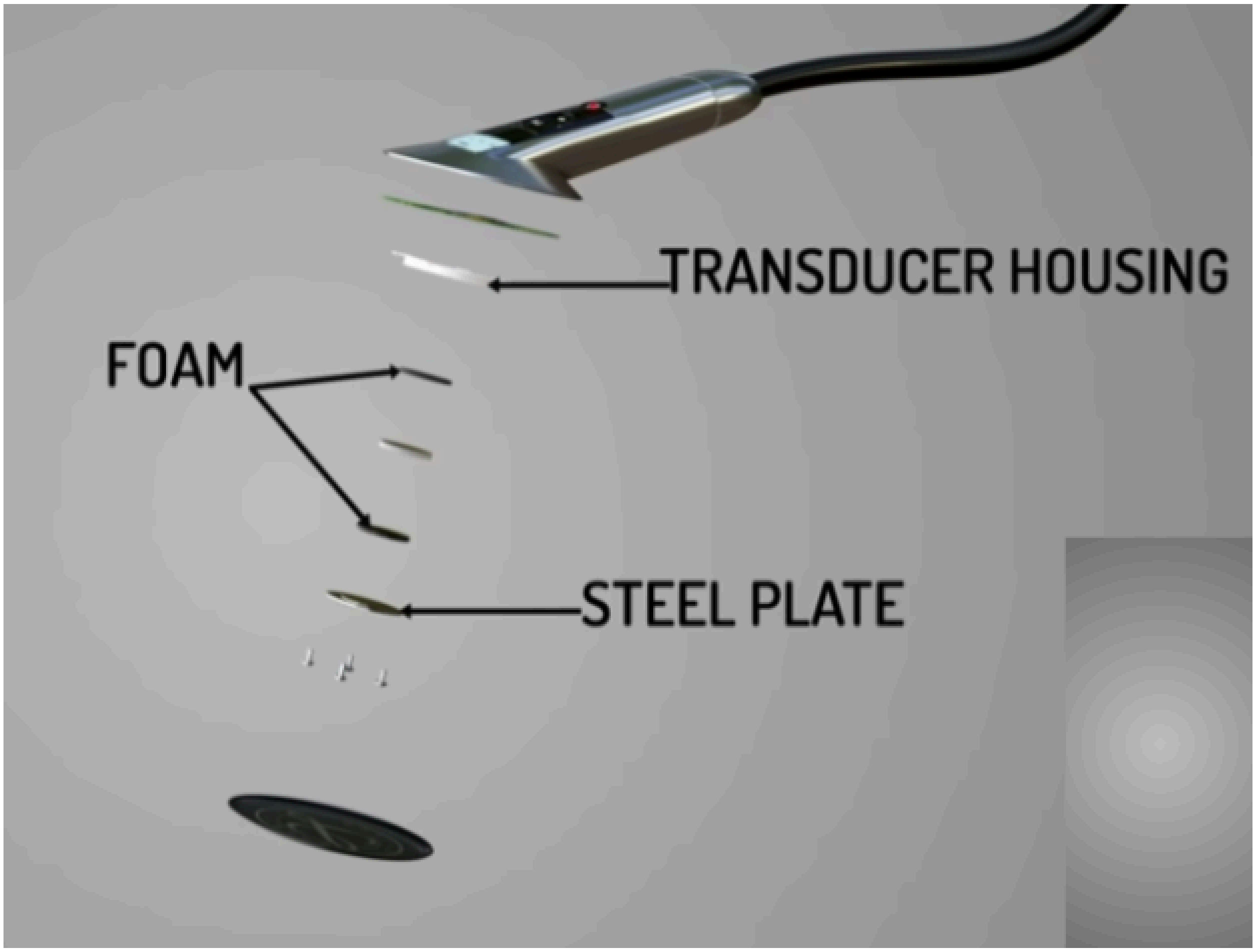
In advanced electronic stethoscopes, the amplified signals are processed using digital signal processing (DSP) technology.

This processing ensures that the sounds are as clear and accurate as possible.

4. Sound Transmission and Output

After processing, the enhanced audio signals are transmitted to the healthcare provider through headphones or an external speaker.

In summary, an electronic stethoscope operates by converting body sounds into electrical signals using sensitive sensors, amplifying and processing these signals, and transmitting the enhanced audio.



The steel are used to improve the performance and corrosion durability.

The transducer housing in an electronic stethoscope, made of steel or other durable materials, protects the sensitive internal components and ensures precise sound capture.

Piezoelectric Transducers

Piezoelectric transducers are devices that convert mechanical energy into electrical energy and vice versa.

Imagine a small piece of quartz embedded in a medical ultrasound probe. When a voltage is applied to the quartz, it deforms slightly. This deformation generates ultrasonic waves that travel through the body. When these waves contact with different tissues, they are reflected back to the probe, causing the quartz to deform again and generate an electrical signal. This signal is then processed to create an image of the internal structures of the body.

Types of Piezoelectric Transducers :

1. Ultrasonic Transducers: These are widely used in medical imaging and distance measurement. In medical applications, ultrasonic transducers emit high-frequency sound waves and receive the echoes that bounce back from internal organs, creating detailed images for diagnosis.

2. Accelerometers: These transducers measure acceleration forces. For instance, in a smartphone, a piezoelectric accelerometer detects the orientation of the device and adjusts the screen display accordingly.

3. Hydrophones: Used in underwater applications, hydrophones detect sound waves traveling through water.

4. Piezoelectric Buzzers: These used in consumer electronics for sound generation. They work by converting electrical signals into audible sound. For example, the beep sound when you press a button on a microwave oven is produced by a piezoelectric buzzer.

Advantages of Piezoelectric Transducers

1. High Sensitivity:

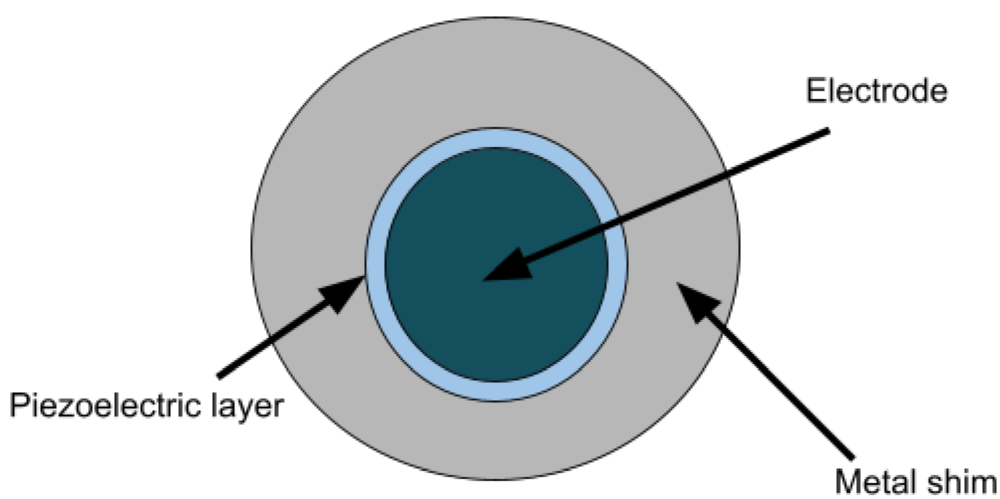
2. Wide Frequency Range: They can operate over a wide range of frequencies, from very low to ultrasonic.

3. Durability and Reliability:

4. Compact Size: These transducers can be made very small, which is beneficial.

Integration with Renewable Energy Systems: Combining piezoelectric energy harvesting with other renewable energy sources could create hybrid systems that are more efficient and reliable.

Biomedical Innovations: Developing new biomedical devices that utilize piezoelectric transducers for better diagnostics and treatment options.



Piezoelectric Technology in Aerospace

These technologies can transform how aircraft and spacecraft operate, from improving structural health monitoring to improving vibration control and energy harvesting.

Structural Health Monitoring:

Safety is the most important aspect in any filed. Piezoelectric transducers play a vital role in structural health monitoring (SHM) systems, which are essential for maintaining the safety.

In SHM systems, piezoelectric sensors are embedded in various parts of an aircraft's structure, such as the wings and control surfaces. These sensors are highly sensitive to even small and fast changes in stress, strain, or vibrations within the material.

When stress is applied to the aircraft structure, the piezoelectric sensors generate an electrical signal proportional to the amount of strain experienced. This signal is then analyzed to detect any anomalies, such as cracks.

Vibration Control and Noise Reduction:

Aircraft and spacecraft are subject to vibrations during operation, which can lead to structural fatigue and increased noise levels.

Piezoelectric materials are suited for active vibration control and noise reduction, contributing to quieter and more durable aerospace vehicles.

Energy Harvesting:

Piezoelectric materials can convert mechanical vibrations and pressure into usable electrical energy.

Piezoelectric energy harvesters can be embedded in aircraft structures to use the mechanical energy generated by vibrations, wing flexing, or even pressure changes during flight.

This energy can then be used to power low-energy devices on the aircraft, such as wireless sensors or monitoring systems, reducing the need for traditional power sources and contributing to overall energy efficiency.

Piezoelectric technology offers a wide range of applications in aerospace. It is specially useful in the SHM system as it focuses on the safety of the rocket and allows the measurement of cracks or slight damages to the rocket.

Acknowledgement

This is to express my deepest gratitude to all the people who supported me in every aspect. I would like to take this opportunity to thank my teachers and specially my family for supporting me and standing with me for my highs and lows.

I would also like to express my heartful gratitude for Harvard University for allowing me to learn about energy and Thermodynmics through the edx platform and also to Massachusetts Institute of Technology for providing the electronic and circuits course through the edx platform.

Finally, to everyone who contributed to this endeavor, thank you for your support and encouragement.

About The Book

This book focus on the basics of electronics and circuit including the methods of analysis for linear and non-linear circuit elements. It also focuses on the digital and the anoluge world and allows to gain an understanding of how digital communication system works.

The second part is Energy and Thermodynamics. This demonstrates the concepts similar to university chemistry with a blend of physics towards the beginning. Vital topics such as the four trajectories and enthalpy is also covered. It also focus on concepts such as the first law of thermodynamics and how the gasoline engine works.

The final part focus on the Piezoelectric technology. This is an incredible form of technology in the modern world and there are interesting applications that made out of this technology.